**On some theoretic aspects of fuzzy subsets**

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**Abstract.** This article proposes some important properties of -fuzzy power set of a finite set. Here, we take a non-empty finite set and an ordered subset of the closed interval, then the set of mappings from to denoted by is defined as -fuzzy power set. Considering the disjunctive union ‘’ operations between fuzzy subsets of , the structure forms a groupoid, which is defined as special fuzzy groupoid. Later, we try to introduce the product of special fuzzy groupoids and their properties.

**Keywords: -f**uzzy power set, disjunctive union, special fuzzy groupoid, product of special fuzzy groupoids

1. **Introduction**

The concept of fuzzy sets was introduced by Zadeh [13] is applied to many mathematical branches. This concept was adapted by Goguen [12] and Sarchez [11] to define and study fuzzy relations. -fuzzy sets are the generalization of fuzzy sets was first considered in 1967 by Joseph Goguen. Since then studies on -fuzzy sets have been carried out by many researchers. Rosenfeld [14] inspired the fuzzification of algebraic structures and introduced the notion of fuzzy subgroups.

Algebraic structures play an important role in mathematics with wide ranging applications in many disciplines such as computer science, information technology, coding theory and so on. There are many researches which had given emphasis on the algebraic structures of the fuzzy sets [2, 6, 15, 16, 17].

Again, the concept of an -set creates applications in many disciplines like mathematics, computer science etc. In real processes there are often situations when the object is a set of “Washed Out” boundaries in the sense that a given element may belong to a given element may belong to agiven set to a greater or lower degree.

This provides sufficient motivation to researchers to review various concepts and results from valued fuzzy sets and formations of algebra from them. In this article we represent -fuzzy power set as special fuzzy groupoid with the help of disjunctive union operation of fuzzy subsets. Throughout the article, different properties of the special fuzzy groupoid are discussed.

1. Preliminaries

This section lists some basic definitions and results which will be used in this article:

***Fuzzy Subsets***: Let  be a space of objects and  be a generic element of and be a classical set. A fuzzy subset,  of  is characterized by a membership function  such that the number  in the unit interval  is interpreted as the degree of membership of element to the fuzzy subset .

***Groupoid***: A non-empty set of elements is said to form a groupoid if in is defined a binary operation called the product denoted by such that for all . It is important to mention have that the binary operation defined on the set  need not be associative.

1. **The fuzzy power set and Special fuzzy Groupoid**

The set of all fuzzy subsets of is called the fuzzy power set of denoted by . Since there is infinite numbers of values in , the fuzzy power set also have infinite numbers of fuzzy subsets or elements. That is why, in this article we take a non-empty finite set and an ordered subset  of such that , where is any positive integer greater than 1. Then the set of all fuzzy subsets obtained from the mappings from to is called the -**fuzzy power set** denoted by . The total numbers of elements or fuzzy subsets in =. For better identification we denote the fuzzy subsets of as 0,1,2…. as follows:





Hence the -fuzzy power set, , where the element ‘0’ is the empty fuzzy subset which is also the infimum and  or  is the universal fuzzy subset which is also the supremum in .

* 1. **Disjunctive Union operation of fuzzy subsets**

The disjunctive union of two fuzzy subsets and of denoted by  and defined as:



**Proposition 1**: The -fuzzy power set is closed under the disjoint union operation.

Proof Let, be the-fuzzy power set. Let, and be any two elements of ; where 



Hence, is closed under .

**Proposition 2**: The special fuzzy groupoid is commutative with respect to the disjunctive union  operation.

Proof: Let, and for  be any two elements of .

Then,



Since,-fuzzy power set is closed under  which is proved in proposition 1. Hence with the operation  is a groupoid and it is defined as the special fuzzy groupoid.

1. **Product of Special fuzzy Groupoids**

The product  of two special fuzzy groupoids  and  is defined by the Cartesian product of the fuzzy subsets of  and . The product  of  and consists of the pair ; such that  is an element of  and  is an element of . Therefore, if the special fuzzy groupoid consists of -fuzzy subsets and  consists of -fuzzy subsets then their product  will consists of  numbers of such pairs of fuzzy subsets.

The meet (fuzzy intersection) and join (fuzzy union) of two pairs in  is formed as:



Here,  and  are the fuzzy intersection and fuzzy union in , while  and  are the fuzzy intersection and fuzzy union in .

The complement of a pair  in  is formed as:



where and are the complements of and in the special fuzzy groupoidand respectively. Under all the operation , the product  also forms a special fuzzy groupoid The fuzzy subsets of the product  of special fuzzy groupoids  and  satisfies some properties. i.e if  be three element (in this case three pairs) of  then:

1. **The laws of forming Complement**

 a)  b) 

1. **Commutative laws**:

 a)  b) 

1. **Distributive laws:**

 a)  b) 

1. **Identity laws**:

 a) b) 

1. **Complement laws**:

 a)  b) 

1. **Involution law**: 

For instance, here is the proof that the commutative laws and distributive laws are shown below:

*Proof of commutative laws*: Commutative law for join holds by the fuzzy subsets in :

 [by the definition of join in ]

 = [commutative law for join is valid in  and ]

 =

Similarly, the commutative law for meet hold in :

 [by definition of join in ]

 = [commutative law for meet is valid in  and ]

 =

*Proof of Distributive law:* 

Similarly, the distributive law for join can also be proved.

**Theorem 4.1**:

The product of two special fuzzy groupoids is also a special fuzzy groupoid.

*Proof*: The product with operation disjunctive union is also a special fuzzy groupoid since it is closed.

**Example 1:** Let, be a finite set and  and are the sets of membership values, Then the mappings from  into are the fuzzy subsets and we can obtain 43=64 elements or fuzzy subsets. Similarly, the mappings from  into are the fuzzy subsets and we can obtain 33=27 elements or fuzzy subsets as shown below:

and



 



So,  and  are two special fuzzy groupoids with respect to the fuzzy disjunctive operation. Hence, there product  is also a special fuzzy groupoid that will contain elements follows:



It also a special fuzzy groupoid with the operation .

1. **Conclusions**

The proposed work presented some important properties of -fuzzy power set. Considering the disjunctive union ‘’ operations between two fuzzy subsets of the-fuzzy power set , it forms a groupoid, which is defined as special fuzzy groupoid. Throughout this article different properties of the -fuzzy power set and special fuzzy groupoid are studied. We hope the results established in this article would find some applications and enrich the theory of fuzzy algebra.

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