**A STUDY ON GENERALIZED RICCI SOLITONS ON LP-SASAKIAN MANIFOLDS**

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**Abstract:** The LP-Sasakian manifold was investigated in this chapter. At first we introduced historical background of the concern manifold. Next some rudimentary facts and related properties of LP-Sasakian manifold are discussed. After that LP-Sasakian manifold concerning generalized Ricci soliton is studied and investigate main result in the form of theorem that is LP-Sasakian manifold of odd dimension satisfying the generalized Ricci soliton equation is an Einstein manifolds.

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***Key Words:*** *LP-Sasakian manifold, Lorentzian para-Sasakian Manifold, Lorentzian Metric, Riemannian manifold, Ricci Soliton, Einstein Manifold.*

1. **INTRODUCTION**

An developing area of contemporary mathematics is the geometry of contact manifolds. The mathematical formalisation of classical mechanics has given way to the concept of contact geometry [7]. K- contact manifolds and sasakian manifolds are two significant kinds of contact manifolds [1], [20]. There are various researchers that have analyzed K-contact and Sasakian manifolds ( [21], [3], [4], [11], [19], [23]) and many others.

The concept of the LP-Sasakian manifold was initially introduced by Matsumoto [13]. Mihai and Rosca defined the same notion independently in [16]. This type of manifold is also discussed in ([14, [22]). A complete regular contact metric manifold  carries a K-contact structure, which is described in terms of almost kaehler structureof the base manifold's . If the base manifold in this case is Kaehlerian, the K-contact structure is Sasakian. If  is only almost Kaehler then is only K-contact [1]. Recent research in [12] has demonstrated the existence of K-contact manifolds that are not Sasakian. Even yet, Sasakian and contact structures are intermediated by K-contact structures. Numerous writers, including [3, [4], [9], [19], [21], [23], have researched K-contact manifolds.

Let us consider function on , then

* 1. 
  2. 

for all smooth vector fields . For a smooth vector field we have ([15],[18])

* 1. 

The generalized Ricci soliton equation in a Riemannian manifold is described by [18]

(1.4) 

where  is the lie derivative of, defined by

(1.5) 

for all vector fields and. For different values of equation (1.4) is a generalization of killing equation  , for soliton , homotheties , , vaccum near-horizon geometry equation  etc. We suggest the reader for further information ([2], [5], [6], [10], [18]).

If , then the equation for the generalized Ricci soliton is [8]

(1.6) 

The work in present Chapter motivated by [8], for the fact that relationship between LP-Sasakian and K-contact manifold, so we studied (2n+1)-dimensional Lorentzian para- Sasakian manifold over generalized Ricci soliton.

**2. PRELIMINARIES**

A (2n+1)-dimension differentiable manifold will be LP-Sasakian manifold [13] [16], if it aquire the (1,1) tensor field , vector field,  is a 1 form on M , lorentzian metric g, accept [14],[17]

(2.1) 

(2.2) 

(2.3) 

(2.4) 

(2.5) 

(2.6) 

(2.7) 

as any vector fields, on .

Additionally, If a manifold's Ricci tensor has the following form given below, it becomes an Einstein manifold:

(2.8) 

for vector fields .

Substituting  in (2.6) and then (2.4) and (2.2), we get

(2.9) 

Take in account (2.9) , we have from (2.8)

(2.10) 

similarly from (2.10) we infer

(2.11) 

**3. GENERALIZED RICCI SOLITON ON LP-SASAKIAN MANIFOLD**

**Theorem 3.1.** Let be a LP-Sasakian manifold then

(3.1) 

for smooth vector fields with orthogonal to .

**Proof:** It is known that

(3.2) 

using (1.5) in (3.2) yields

(3.3) 

by definition of Riemannian curvature tensor, from (3.3) it follows that

(3.4) 

using (2.4) in (3.4) and with orthogonal to , we get

(3.5) 

so, (3.4) may be expressed as

(3.6) 

**Lemma 3.2:** Let  be a Riemannian manifold and let  be a smooth function. Then [15]

(3.7) 

for every vector field .

**Theorem 3.2:** Let is a LP-Sasakian manifold which accept the generalized Ricci soliton equation. Then

(3.8) 

**Proof:** Using (2.6) we have

(3.9) 

Making use of (1.6) and (3.9) implies

(3.10) 

The lemma thus follows from (3.5) and (1.6), which gives the Hessian definition.

Next, Suppose that is orthogonal to . From Lemma 3.1, and taking , we get

(3.11) 

by Lemma (3.2) and above equation, we obtain

(3.12) 

since and from equation (2.10), we obtain

(3.13) 

Note that, from equation (2.3) , we have it implies . Applying the Lie derivative to the generalised Ricci soliton equation (1.6) and the aforementioned fact:

(3.14) 

Using (3.13), (3.14) and Lemma (3.2) we infer that

(3.15) 

According to Lemma 3.2 we have

(3.16) 

by equation (3.15) and (3.16), we obtain

(3.17) 

Which implies 

Provided. Therefore  is parallel to . Hence as is nowhere integrable, that is, is a constant function. Thus the manifold is an Einstein one follows from (1.6), so we concluded that

**Theorem 3.3:** If is a odd-dimensional LP-Sasakian manifold that satisfies the generalized Ricci soliton equation with. Then  has a constant value. Additionally, manifold is an Einstein manifold if . The lemma thus follows from (3.5) and (1.6), which gives the Hessian definition.

**References**

[1] Blair, D.E., Contact manifolds in Reimannian geometry, Lecture notes in Math. 509, Springer-Verlag., 1976.

[2] Chrusciel, P.T., Reall, H.S., Tod, P.: On non-existence of static vacuum black holes with degenerate components of the event horizon. Classical Quantum Gravity, 23, 549-554 (2006).

[3] De, U.C. and Biswas, S., On K-contact η-Einstein manifolds. Bull. Soc. Math. 16 (1990), 23-28.

[4] De, U.C., De, A., On some curvature properties of K-contact manifolds. Extracta Math. 27 (2012), 125-134.

[5] Deshmukh, S., Aloden, H.: A note on Ricci soliton. Balkan J. Geom. Appl. 16 , 48-55 (2011).

[6] Deshmukh, S.: Jacobi-type vector fields on Ricci solitons. Bull. Mathematique de la Societe des Sciences Mathematiques de Roumanie Nouvelle Series. 103, 41-50 (2012).

[7] Geiges, H., A brief history of contact geometry and topology. Expo. Math. 19 (2001), 25-53.

[8] Gopal Ghosh and Uday Chand De, Generalized Ricci solitons on K-contact manifolds, MATHEMATICAL SCIENCES AND APPLICATIONS E-NOTES, 8 (2) 165-169 (2020) c MSAEN, HTTPS://DOI.ORG/10.36753/MATHENOT.683478.

[9] Guha, N., De, U.C.: On K-contact manifolds. Serdica-Bulgaricae Math. Publ. 19, 267-272 (1993).

[10] Jezierski, J.: On the existance of Kundts metrics and degenerate (or extremal) Killing horizones. Classical Quantum Gravity, 26, 035011, 11pp (2009).

[11] Jun, J-B., Kim, U.K., On 3-dimensional almost contact metric manifolds. Kyungpook Math. J. 34 (1994), 293-301.

[12] Koufogiorgos, T.: Contact metric manifolds. Ann. Global Anal. Geom. 11, 25-34 (1993).

[13] Matsumoto, K. On Lorentzian paracontact manifolds. *Bull. of Yamagata Univ. Nat. Sci.*,

1989, 12, 151–156.

[14] Matsumoto, K. and Mihai, I. On a certain transformation in a LP-Sasakian

manifold. *Tensor, N. S.*, 1988, 47, 189–197.

[15] Mekki, M.E., Cherif, A.M.: Generalised Ricci solitons on Sasakian manifolds. Kyungpook Math. J. 57 677-682 (2017).

[16] Mihai, I. and Rosca, R. *On Lorentzian P-Sasakian Manifolds, Classical Analysis*. World

Scientific, Singapore, 1992, 155–169.

[17] Mihai, I. A.A. Shaikh and Uday Chand De, On LP-Sasakian Manifolds.

[18] Nurowski. P., Randall, M.: Generalised Ricci solitons. J. Geom. Anal. 26, 1280-1345 (2016).

[19] Prasad, R., Srivastava, V., On ϕ-symmetric K-contact manifolds. IJRRAS 16 (2013), 104-110.

[20] Tanno, S., Isometric immersions of Sasakian manofolds in spheres. K¯odai Math. Sem. Rep. 21 (1969), 448-458.

[21] Tarafdar, D. and De, U.C., On K-contact manifolds. Bull. Math. de la Soc. Sci. Math de Roumanie 37 (1993), 207-215.

[22] Tripathi, M. M. and De, U. C. Lorentzian almost paracontact manifolds and their

submanifolds. *J. Korea Soc. Math. Educ. Ser. B: Pure Appl. Math.*, 2001, 8, 101–105.

[23] Yildiz, A., Ata, E.: On a type of K-contact manifolds. Hacettepe J. Math. Stat. 41, 567-571 (2012).