**EFFECTS OF WALL PROPERTIES AND HEAT TRANSFER ON THE PERISTALTIC TRANSPORT OF FOOD BOLUS THROUGH OESOPHAGUS – A MATHEMATICAL MODEL**

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**ABSTRACT**

A mathematical model is constructed to study the effect of heat transfer and elasticity of flexible walls in swallowing of food bolus through the oesophagus. The food bolus is supposed to be viscous fluid and the geometry of wall surface of oesophagus is considered as peristaltic wave. The expressions for temperature field, axial velocity, transverse velocity and stream function are obtained under the assumptions of low Reynolds number and long wavelength. The effects of thermal conductivity, Grashof number, rigidity, stiffness of the wall and viscous damping force parameters on velocity, temperature and stream function have been studied. It is observed that when the magnitude of Grashof number, thermal conductivity, rigidity and stiffness of the wall increase, the axial velocity of the channel increases where as the increase in viscous damping force reduces the axial velocity. It is found that the size of the left trapped bolus decreases while the size of the right trapped bolus increases with increasing heat source.

**Keywords** : Peristaltic transport, Oesophagus, Food bolus, Viscous fluid.

**1. INTRODUCTION**

Peristaltic transport is a form of material transport induced by a progressive wave of area contraction or expansion along the length of a distensible tube. The mechanism of peristalsis has important applications in understanding many transport processes through physiological systems under peristaltic motion. In the living body peristalsis is involved in the swallowing food through the esophagus, the transport of urine from the kidney to the bladder through the ureter and in the motion of the chyme in the small intestine. It is believed that peristalsis is also involved in some circulatory systems such as the lymph flow in some lymphatic vessels (Hewson 1774 and Hall *et al*. 1965) and in the vasomotion of the small blood vessels (Nicoll 1956). In these cases, the peristaltic contraction of vessel wall is more or less like that of a travelling smooth wave. Peristaltic flow of a single fluid through an infinite tube or channel in the form of sinusoidal wave motion of the tube wall is investigated by Burns and Parks (1967), Hanin (1968) and Shapiro *et al.* (1969) and several others. Further, many of the physiological fluids are known to be non-Newtonian. Some important analytical studies on peristaltic motion of non-Newtonian fluids available in the literature are Devi and Devanathan (1975); Shukla and Gupta (1982); Srivastava and Srivastava (1984); Usha and Rao(1995), Vajravelu *et al.* (2005a, 2005b); Hayat and Ali(2008); Hayat *et al.* (2010a, 2010b) etc.

An interesting fact is that in oesophagus, the movement of food is due to peristalsis. The food moves from mouth to stomach even when upside down. Oesophagus is a long muscular tube commences at the neck opposite the long border of cricoids cartilage and extends from the lower end of the pharynx to the cardiac orifice of the stomach. The swallowing of the food bolus takes place due to the periodic contraction of the oesophagal wall. Pressure due to reflexive contraction is exerted on the posterior part of the bolus and the anterior portion experiences relaxation so that the bolus moves ahead. The contraction is practically not symmetric; yet it contracts to zero lumen and squeezes it marvelously without letting any part of the food bolus slip back in the opposite direction. Any imbalance may result into a retrograde motion. The influence of wall properties on the Poiseuille flow under peristalsis is studied by Mittra and Prasad (1973). Mishra and Pandey (2001) analyzed a mathematical model for the oesophagal swallowing of a food-bolus. A new model for study the effect of wall properties on peristaltic transport of a viscous fluid has been investigated by Mokhtar and Haroun (2008). Radhakrishnamacharya and Srinivasulu(2007) studied the influence of wall properties on peristaltic transport with heat transfer.

Since bioheat is currently considered as heat transfer in the human body, the mode of bioheat transfer in tissues has been attracted by the biomedical engineers. Infact the heat transfer in human tissues involves complicated processes such as heat conduction in tissues, heat transfer due to perfusion of the arterial-venous blood through the pores of the tissue, metabolic heat generation and external interactions such as electromagnetic radiation emitted from cell phones. Hence in fluid mechanics, it is found that temperature plays an important role during the fluid flows. More recently Srinivas *et al.* (2009) studied the effect of slip, wall properties and heat transfer on MHD peristaltic transport.

Motivated by this, we consider a mathematical model to study the effect of wall properties and heat transfer on swallowing the food bolus through the oesophagus. The results are analyzed for different values of parameters namely Grashof number, thermal conductivity, rigidity, stiffness and viscous damping forces of the channel wall.

**2. MATHEMATICAL FORMULATION**

Consider peristaltic transport of an incompressible Newtonian fluid in a channel with flexible walls induced by sinusoidal wave trains propagating with constant speed c along the channel walls.

(1)

where , , a , , and c represent transverse vibration of the wall, axial coordinate, time, half width of the channel, amplitude of the wave, wavelength and wave velocity respectively.

The governing equations of motion of incompressible viscous fluid with heat transfer are given as

(2)

(3)

(4)

(5)

where , , , , , , g , , T , , K and denote the fluid density, axial velocity, transverse velocity, transverse coordinate, pressure, fluid viscosity, acceleration due to gravity, coefficient of linear thermal expansion of fluid, temperature, specific heat at constant pressure, thermal conductivity and constant heat addition **/** absorption.

The temperatures at the center line and the wall of the peristaltic channel are given as

= at (6)

where the temperature at centre is line and is the temperature on the wall of peristaltic channel.

The governing equation of motion of the flexible wall may be expressed as

where is an operator, which is used to represent the motion of stretched membrane with viscosity damping forces such that

Continuity of stress at and using momentum equation, yield

(7)

Here is the elastic tension in the membrane, is the mass per unit area, C is the coefficient of viscous damping forces.

Introducing the following non – dimensional quantities,

(8)

where , , , , , , and are wave number, length of the channel, stream function, volume flow rate, Reynolds number, Grashof number, dimensionless temperature, dimensionless heat source/sink parameter and Prandtl number

in equations (1 – 8), we finally get

(9) (10)

(11)

(12)

(13)

(14) (15)

**3. SOLUTION OF THE PROBLEM**

Under the assumptions of long wavelength and low Reynolds number, equations (9 -15)reduce to

(16)

(17)

(18)

(19)

(20)

(21)

(22)

The following boundary conditions are imposed on the governing equations to model the problem under consideration**:**

i.e., the regularity condition (23)

i.e., the no slip condition (24)

i.e., the absence of transverse velocity (25)

Equation (18)shows that P is not a function of. Now on differentiating equation (17**)** with respect to , the compatibility equation as follows

(26)

Equation (22)gives (27)

The closed from solution for equations (17) and (20) with the boundary conditions (21),(23) and (24) is

(28)

(29)

Integrating the continuity equation with respect to , and using equation (29) and the boundary condition (25), we obtain transverse velocity as

(30)

Stream function can be obtained by integrating eqn (29) and using the condition at . It is given by

(31)

**4. RESULTS AND DISCUSSIONS**

In order to observe the quantitative effects of various parameters involved in the analysis, the velocity, temperature and stream functions are calculated for various values of these parameters. Computer codes were developed for the numerical evaluations of the analytical results and some important results are displayed graphically in Fig(1)-(11). From Fig.(1) and (2) it is observed that increase in thermal conductivity β and Grashof number Gr results in increase of velocity in the channel.

|  |  |
| --- | --- |
| 0.0  0.2  0.4  0.6  0.8  1.0  5.0  4.0  u  3.0  2.0  1.0  0.0  1.2  1.4  β=0.0  β=3.0  β=6.0  β=9.0  η | 0.2  0.4  0.6  0.8  1.0  1.2  1.4  0.0  0.5  1.5  2.5  3.5  u  3.0  2.0  1.0  0.0  Gr=0.0  Gr=1.0  Gr=2.0  Gr=3.0  η |
| Figure 1: Velocity Profiles for different β  ( | Figure 2: Velocity Profiles for different Gr  ( |

Fig. (3) displays the effect of rigidity parameter in the presence of stiffness and viscous damping force. It is noticed that the velocity increases with increase in rigidity parameter. A similar observation is made for different values of in the presence of other parameters i.e., rigidity and viscous damping force which is shown in Fig. (4).

|  |  |
| --- | --- |
| 0.0  0.2  0.4  0.6  0.8  10  0  1.0  20  25  30  35  u  15  5  =0.5  =1.0  =1.5  η | 0.0  0.2  0.4  0.6  0.8  1.0  0  5  10  15  20  25  u  =0.1  =0.5  =1 |
| Figure 3: Effect of on velocity  ( | Figure 4: Effect of on velocity  ( |

Fig. (5) is plotted to see the influence of viscous damping force on velocity distribution in the presence of rigidity and stiffness. One can observe that the velocity decreases with the increase in.

The variation in temperature for various values of thermal conductivity is shown in Fig.(6). The temperature increases with the increase in .

|  |  |
| --- | --- |
| 0.0  0.2  0.4  0.6  0.8  1.0  0  2  4  6  8  10  12  u  =0.1  =0.5  =0.9  η | 0.2  0.4  0.6  0.8  1.0  1.2  1.4  0.0  0.5  1.0  1.5  2.0  0.0  β=0.0  β=2.0  β=4.0  β=6.0  θ  η |
| Figure 5: Effect of on velocity  ( | Figure 6: Temperature profiles for different |

An interesting phenomenon of peristalsis is trapping in which streamlines split to trap a bolus in the wave frame. The effect of thermal conductivity on trapping is analyzed in Fig.(7). It can be concluded that the size of the trapped bolus in the left side of the channel decreases when increases where as it has opposite behavior in the right hand side of the channel.

The influence of Grashof number on trapping is analyzed in Fig.(8). It reveals that the size of the left trapped bolus decreases with increase in Gr where as the size of the right trapped bolus increases with increase in Gr. The effect of on trapping can be seen in Fig.(9). We notice that the size of the bolus increases with increase in . Fig. (10) shows the influence of on trapping. One can observe that the size of the trapped bolus decreases with increase in , stiffness of the wall. The effect of on trapping is shown in Fig. (11). It is concluded that the size of the left bolus decreases where as the right bolus increases with increase in . Furthermore, it is observed that more trapped bolus appears with increase in .

**5. CONCLUSIONS**

The present study deals with the combined effect of heat transfer and wall properties on the peristaltic transport of a viscous fluid in a two dimensional channel. We obtained the analytical solution of the problem under long wave length and low Reynolds number assumptions. The results are analyzed for different values of pertinent parameters namely Grashof number, thermal conductivity, rigidity, stiffness and viscous damping forces of the channel wall. Some of the interesting findings are

1. The axial velocity increases with the increase in . Further, the axial velocity decreases with increase in .
2. The coefficient of temperature increases with increasing values of thermal conductivity.
3. The volume of the trapped bolus increases with increase in . Moreover, more trapped bolus appears with increase in .

-0.4

0.2

0.0

0.2

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.0

0.6

0.4

0.0

0.2

0.6

0.8

1.0

1.2

1.4

0.4

0.6

0.4

0.2

0.0

-0.2

-0.4

(a) (b)

-0.4

-0.2

0.0

0.2

0.4

0.6

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.0

(c)

Figure 7 : Effect of on Trapping (a) (b) (c)

(

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

-0.4

-0.2

0.0

0.2

0.4

0.6

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.6

0.4

0.2

0.0

-0.2

-0.4

1. (b)

-0.4

-0.2

0.0

0.2

0.4

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.0

0.6

(c)

Figure 8: Effect of on Trapping (a) (b) (c)

(

-0.4

-0.2

0.0

0.2

0.4

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.6

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.6

0.4

0.2

0.0

-0.2

-0.4

1. (b)

0.6

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.4

0.2

0.0

-0.2

-0.4

(c)

Figure 9 : Effect of on Trapping (a) (b) (c)

(

0.6

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.4

0.2

0.0

-0.2

-0.4

1.4

1.2

1.0

0.8

0.6

0.4

0.2

0.0

-0.4

-0.2

0.0

0.2

0.4

0.6

(a) (b)

-0.4

-0.2

0.0

0.2

0.4

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.6

(c)

Figure 10: Effect of on Trapping (a) (b) (c)

(

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.6

0.4

0.2

0.0

-0.2

-0.4

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.6

0.4

0.2

0.0

-0.2

-0.4

1. (b)

0.6

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

0.4

0.2

0.0

-0.2

-0.4

(c)

Figure 11: Effect of on Trapping (a) (b) (c)

(

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