**5 D Emergent Kerr Tunneling Vacua**

1. **Introduction**

The bulk/boundary correspondence [1,2] is a powerful tool to establish quantum gravity via a gauge theory. It sheds light how the bulk quantum gravity theory may emerge from the gauge boundary theory. In fact, an emergent gravity has been a recent topic of great interest in the folklore of theoretical physics [3,4].

As per the bulk/boundary correspondence, these non-linear U(1) gauge theories are describe on D-brane. In 5 dimensions, we may replace the one form U(1) gauge field by a two form field. The local degrees of a two form have been exploited on a D4-brane to construct a geometric torsion dynamics in a non-perturbation theory of quantum gravity. In fact a geometric torsion has been constructed through a modification of the covariant derivative in presence of two form connections in a non-linear gauge theory. The iterative corrections in two form gauge con-nection lead to an exact description in a perturbative gauge theory. This, in turn, may be viewed as a non-perturbative geometric construction in a second order formalism. Apriori, a geometric torsion breaks the U(1) gauge invariance in the gauge theory. However the gauge invariance is restored in a generalized curvature theory with the help of an emergent metric fluctuation. The local degrees of two form sources the fluctuations and they incorporate the notion of a dynamical space–time curvature on a D4-brane. The emergent space–time may also be viewed through a two form gauge connection which takes an analogues form to the Christoffel metric connection in Riemannian geometry. In a gauge choice for a two form, leading to a non-propagating torsion, the emerging fourth order curvature tensor reduces to a Riemannian curvature [5].

1. **Mathematical Formulation: Gauge theoretic Curvature**

In 5 dimensions the U(1) dynamics in presence of a constant background are described by the following action

S=-

Here is a dimensional constant that represents the gauge coupling, “g” is the constant background metric and F is the U(1) gauge field strength defined as . , is the one form gauge potential. In 5 dimensions, the U(1)gauge field dynamics can be re-expressed in terms of a two form field via Poincare duality. The modified gauge theoretical description by a two form describes a non-linear charge. The new action is

S=-

Where again is a dimensional constant and provides the coupling in the two form theory, H is the field strength corresponding to the two form defined as .

In 5 dimension, we may now explore this torsion term to construct a space time curvature scalar as the dynamical degrees of freedom of the two matches exactly[6-9]. Hence an irreducible scalar underlies a geometric torsion , which is basically governed by a gauge theoretic torsion. Also, this modification of gauge torsion to geometric torsion requires a the redefinition of the covariant derivative in the theory, with a completely anti-symmetric connection. Hence the new covariant derivative may be defined as

 (1)

Formally the geometric torsion can also be expressed as the geometric torsion describing the second order formalism with all order corrections in ina gauge theory

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This geometric torsion is now utilise to construct a curvature tensor in 5 dimensions by taking the commutation relation of covariant derivative in the theory, so the effective curvature tensor obtained is defined as

 (2)

One can now construct the irreducible curvature tensor to write the action in 5 D. The above fourth order tensor retains a property of Riemann tensor Rμνλρ by showing anti-symmetricity within a pair of indices, i.e. μ ↔ ν and λ ↔ ρ. However it is not symmetric, under an interchange of a pair of indices, like Riemann tensor. Hence, we may say that the effective curvature constructed in a non-perturbative framework may be viewed as a generalized curvature tensor. It describes the propagating geometric torsion in a second order formalism.

This effective curvature constructed in the second order formalism preserves the gauge invariance of . As a result the, the space time fluctuations are given as

Where C is a dimensional constant. Also, the effective curvature tenor may be written though a geometric field strength which is dual to in 5D. Hence a geometric may be given by

Where B represents the non-dynamical modes of the two form field and already been shown towards the construction of non-linear U(1) gauge theory []. Now these fluctuations due to the dynamics and constant modes of the two form field can be added to generate the effective metric in the theory as

 (3)

Here B represents the contribution of constant(non-dynamical ) modes two form, shows the geometric torsion. As these non-dynamical modes are not unique they may give us a large no of vacua in the theory. The effective curvature may seen to describe a geometric torsion dynamics in 5D [32, 33]. A geometric construction of a torsion in a non-perturbative framework is inspiring and may provoke thought to unfold certain aspects of quantum gravity. Generically, the action may be given by

Here is the dimensional constant in the action and G is the effective metric containing constant and non-constant modes of the two form field as defined in equation (3). Finally is the effective curvature tensor.

1. **Choice of background metric and gauge ansatz:**

In this section we will first look into the choice background metric which is non dynamical and acts as background to the effective curvature tensor . A priori, these quantum modes considered provides extra term to the whole geometry but they soon decouple in the semiclassical limit in order to produce a stable Kerr blackhole solution in 5D. The background for the Kerr black hole is best described as Boyer-Lindquist coordinates which is given by

Where and

The range of the angular coordinates are: (0 <φ<2π, 0<ψ<2π, 0<θ<π). They may be expressed by the cartesian coordinates:

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 and

The above transformation shows ensures a ring singularity about the x-y and w-z plane.

* 1. Ansatz for two form field:

The effective metric considered can now be explored to give quantum blackhole geometry by substituting different form of two form field .[9] In the context, a gauge choice considered is

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Under this choice of ansatz we obtained the Kerr Black Hole Solution in 5 dimensions. This vacua seems to tunnel to a another similar solution with the following set of B field consideration.

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Both the solution in the low energy limit seems to tunnel among themselves. We can also say that the potential considered for constructing the The U(1) gauge theory is not unique.

**Conclusion:**

We started with a generalized curvature theory whose source is a two form in a U(1) gauge theory. We revisited the theory to obtain some of the quantum Kerr vacua in five dimensions. The Kerr geometries were constructed in a nonperturbative framework, underlying a geometric torsion, on a non BPS brane.

In particular, the five dimensional emergent Kerr geometries purely sourced by the background fluctuations in B2 on a non-BPS brane were constructed. The gauge choice freezes the local degrees in torsion, which is described by a five dimensional generalized curvature. A vanishing torsion may alternately be viewed as a vanishing energy momentum tensor in a non-linear U(1) gauge theory. Nevertheless, the background fluctuations in five dimensions may be understood in presence of an electromagnetic field F2, which incorporates local degrees into a non-linear global description. The local F2 may be gauged away in the framework. In addition, the background fluctuations may have their origin in a dynamical two form, and may describe a propagating torison, in higher dimensions. A two form gauge transformations do not make the emergent geometries unique. In fact, there are a very large number of emergent vacua..

References

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