# AL - STRUCTURE ON F- RELATION L - FUZZY TOPOLOGICAL SYSTEMS

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Abstract. We introduced, Algebric structure on L - fuzzy topological system which is described in fuzzy relation . Moreover, the paper Algebric structure on F - relation L - fuzzy topological system provide with cloudless examples.

**AMS Classification:** 03F55, 03E72, 08A72

## Key words : Fuzzy Topology, Fuzzy Relations, TM-Algebra

1. Introduction

Fuzzy concept introduced by L.A.Zadeh [25] at 1965 and developed the Fuzzy relation at 1971. Chang [18], Wong [24] , Lowen [22] and others developed the fuzzy topological spaces. In 2010, Tamilarasi and Manimegalai proposed a new type of algebras known as TM-algebras [23].

we introduced the concept in [1], Fuzzy Topological subsystem on a TM-algebra. We studied in [2] *L−* Fuzzy Topological TM-system. We developed the concept in [3] *L−* Fuzzy Topological TM-subsystem. we studied in [4], [5] Fuzzy Supratopological TM- system, Fuzzy *α−* supracontinuous functions. In this paper, discuss the notion of an AL - Structure on F-Relation L - Fuzzy Topological Systems and investigate some simple properties

1. Preliminaries

**Definition 2.1.** *Let X be a non-empty set. A mapping µ* : *X →* [0*,* 1] *is called a fuzzy set of X.*

**Definition 2.2.** *µ*1 *and µ*2 *are the fuzzy sets of a set X.*

*Then µ*1 *⊂ µ*2 *is defined by µ*1(*x*) *≤ µ*2(*x*) *forall x ∈ X*

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**Definition 2.3.** *µ is a fuzzy set of X. The complement of µ is denoted by µj and defined by µj*(*x*) = 1 *− µ*(*x*) *forall x ∈ X*

**Definition 2.4.** *µ*1 *and µ*2 *are the fuzzy sets of a set X.*

*The union of fuzzy sets µ*1*, µ*2 *is :*

(*µ*1 *∪ µ*2)(*x*) = *MAX {µ*1(*x*)*, µ*2(*x*)*} forall x ∈ X.*

*The intersection of fuzzy sets µ*1*, µ*2 *is :*

(*µ*1 *∩ µ*2)(*x*) = *MIN {µ*1(*x*)*, µ*2(*x*)*} forall x ∈ X.*

**Definition 2.5.** *A family T of fuzzy sets in X, the fuzzy topology which is satisfies the* conditions as

*i) φ, X ∈ T ii) If µ, σ ∈ T then µ ∩σ ∈ T iii) If µi ∈ T for each i ∈ I then ∪Iµi ∈ T*

*where I is an indexing set.*

**Definition 2.6.** *A non-empty set X with a constant 0 and a binary operation ∗ of TM-* Algebra ( *X, ∗,* 0 *) is satisfying the following conditions : i) x∗*0 = *x ii)* (*x∗y*)*∗*(*x∗z*) = *z∗y* for all x, y, z *∈ X.*

# Definition 2.7.

*A fuzzy Subalgebra µ of a TM-Algebra ( X, ∗,* 0 *) if , for all x, y ∈ X,* µ(*x ∗ y*) *≥ min{µ*(*x*) *, µ*(*y*)*}*

**Definition 2.8.** *µ and σ are two fuzzy sets in a fuzzy topological space* (*X, T* )*. σ is* said to be an interior of *µ if µ is a neighbourhood of σ and µ ⊃ σ.*

**Definition 2.9.** *Fuzzy Relation*

*Consider the cartesian product A × B* = *{*(*x, y*) : *x ∈ A, y ∈ B} where A and B in* universal sets U and V correspondingly. A fuzzy relation on *A × B denoted by R or* R(x, y) is defined as the set *R* = *{*(*x, y*)*, µR*(*x, y*) : (*x, y*) *∈ A × B, µR*(*x, y*) *∈* [0*,* 1]*}*

**Definition 2.10.** *The union of fuzzy relations R*1 *and R*2 *is denoted by R*1 *∪ R*2 *is defined by µR*1*∪R*2 (*x, y*) *= min{µR*1 (*x, y*)*, µR*2 (*x, y*)*} ,* (*x, y*) *∈ A × B*

*The intersection of fuzzy relations R*1 *and R*2 *is denoted by R*1 *∩ R*2 *is defined by*

*µR*1*∩R*2 (*x, y*) *= max {µR*1 (*x, y*)*, µR*2 (*x, y*)*} ,* (*x, y*) *∈ A × B*

## AL - STRUCTURE ON F- RELATION L - FUZZY TOPOLOGICAL

SYSTEMS

**Definition 3.1.**

*AL - Structure on F - Relation L - Fuzzy Topological Systems:*

*X, Y are TM-Algebras. R*1(*x, y*)*, R*2(*x, y*) *are the fuzzy relations of X, Y. AL - structrue* on F - relation L - Fuzzy Topological System is a family *T of L - fuzzy subalgebras in* (*X, Y, T* ) *which is satisfies the conditions :*

*i) φ, X ∈ T ii) If µ*(*x, y*)*, σ*(*x, y*) *∈ T then µ*(*x, y*) *∩ σ*(*x, y*) *∈ T iii) If µi*(*x, y*) *∈ T for* each *i ∈ I then ∪Iµi*(*x, y*) *∈ T where I is an indexing L -subalgebra.*

# Example 3.2.

The set *X* = *{*0*,* 1*,* 2*} , Y* = *{*0*,* 1*,* 2*}* with the cayley table

|  |  |  |  |
| --- | --- | --- | --- |
| *∗* | 0 | 1 | 2 |
| 0 | 0 | 1 | 2 |
| 1 | 1 | 2 | 0 |
| 2 | 2 | 0 | 1 |

The fuzzy relations *R*1(*x, y*) and *R*2(*x, y*) are on the sets *X* , *Y* is ,

|  |  |  |  |
| --- | --- | --- | --- |
| *X, Y* | 0 | 1 | 2 |
| 0 | (0*,* 0) | (0*,* 1) | (0*,* 2) |
| 1 | (1*,* 0) | (1*,* 1) | (1*,* 2) |
| 2 | (2*,* 0) | (2*,* 1) | (2*,* 2) |

The F - relations L - subalgebras *µi* : *X →* [0*,* 1]*, i* = 1*,* 2*,* 3 *νi* : *Y →* [0*,* 1]*, i* = 1*,* 2*,* 3 are

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*µ*1(*x, y*) =

*µ*3(*x, y*) =

*ν*2(*x, y*) =

*t*5 *if* (0*,* 0)

*t*3 *if* (0*,* 1)

 *t*1 *if* (0*,* 2)





*t*6 *if* (2*,* 0)

*t*3 *if* (2*,* 1)



*t*1 *if* (2*,* 2)



*t*4 *if* (1*,* 0)

*t*2 *if* (1*,* 1)

 *t*1 *if* (1*,* 2)

*µ*2(*x, y*) =

*ν*1(*x, y*) =

*ν*3(*x, y*) =

*t*8 *if* (1*,* 0)

*t*6 *if* (1*,* 1)

 *t*7 *if* (1*,* 2)





*t*6 *if* (0*,* 0)

*t*4 *if* (0*,* 1)



*t*2 *if* (0*,* 2)



*t*5 *if* (2*,* 0)

*t*4 *if* (2*,* 2)

 *t*2 *if* (2*,* 3)

A family *T* = *{φ, X, µ*1*, µ*2*, µ*3*, ν*1*, ν*2*, ν*3*}* which is satisfying the AL - structure on F - relation L - fuzzy topological system (*X, Y, T* )

# Definition 3.3.

*X, Y are TM-Algebras. R*1(*x, y*)*, R*2(*x, y*) *are the fuzzy relations of X, Y. AL - structrue* on F - relation L - Fuzzy Topological System (*X, Y, T* )*. F - relation L - fuzzy subalgebra N in L - fuzzy topological TM-system, is an F - relation L - fuzzy neighbourhood of an F*

*- relation L - fuzzy subalgebra M if there exist an T-open F- relation L - fuzzy subalgebra*

*D such that M ⊂ D ⊂ N*

*ie M ≤ D ≤ N for all x ∈ X, y ∈ Y*

# Example 3.4.

*µi*(*x, y*) *, i* = 1*,* 2*,* 3 *, νi*(*x, y*) *, i* = 1*,* 2*,* 3 are F - relation subalgebras of the L - fuzzy topological TM-system given in example 3.4

A family *T* = *{φ, X, µ*1*, µ*2*, µ*3*, ν*1*, ν*2*, ν*3*}* is an AL - structrue on F - relation L - Fuzzy

Topological System (*X, Y, T* )*.*

## *µ*2(*x, y*) is F - relation L - fuzzy neighbourhood of an F - relation L - fuzzy subalgebra

*µ*1(*x, y*) for *µ*1(*x, y*) *≤ ν*2(*x, y*) *≤ µ*2(*x, y*)

# Definition 3.5.

*X, Y are TM-Algebras. R*1(*x, y*)*, R*2(*x, y*) *are the fuzzy relations of X, Y. AL - structrue* on F - relation L - Fuzzy Topological System (*X, Y, T* )*. µ*(*x, y*) *is F - relation L - fuzzy* subalgebra in (*X, Y, T* )*. The collection of F - relation L - fuzzy neighbourhood of µ*(*x, y*) *is the set* U(*x, y*) *is said to be a F - relation L - fuzzy neighbourhood system of µ*(*x, y*)*.*

# Definition 3.6.

*X, Y are TM-Algebras. R*1(*x, y*)*, R*2(*x, y*) *are the fuzzy relations of X, Y. AL - structrue* on F - relation L - Fuzzy Topological System (*X, Y, T* )*. The F- relation L - fuzzy interior* of *µ*(*x, y*) *is the union of all F - relation open L - fuzzy subalgebras contained in µ*(*x, y*) *and it is denoted by* (*µ*)*◦*(*x, y*)*.*

*That is* (*µ*)*◦*(*x, y*) *= ∪ {µ*(*x, y*) : *µ*(*x, y*) *⊆ µ*(*x, y*)*, µ*(*x, y*) *∈* (*X, Y, T* )*}*

# Example 3.7.

*µi*(*x, y*) *, i* = 1*,* 2*,* 3 *νi*(*x, y*) *, i* = 1*,* 2*,* 3 are F - relation L - fuzzy subalgebras of the TM-system given in example 3.4

A family *T* = *{φ, X, µ*1*, µ*2*, µ*3*, ν*1*, ν*2*, ν*3*}* is an AL - structrue on F - relation L - Fuzzy Topological System (*X, Y, T* )*.*

The F - relation L - fuzzy interior of (*µ*3)*◦*(*x, y*) = *∪ {µ*1*, ν*2*, ν*4*}* = *µ*1(*x, y*)

# Theorem 3.8.

*X, Y are TM-Algebras. R*1(*x, y*)*, R*2(*x, y*) *are the fuzzy relations of X, Y. AL - structrue* on F - relation L - Fuzzy Topological System (*X, Y, T* )*. F - relation L - fuzzy subalgebra U* (*x, y*) *is open in* (*X, Y, T* ) *if and only if for each F - relation L - fuzzy subalgebra V*(*x, y*) *contained in U* (*x, y*)*, U* (*x, y*) *is F- relation L - fuzzy neighbourhood of V*(*x, y*)*. Proof:*

*AL - structrue on F - relation L - Fuzzy Topological System* (*X, Y, T* )*. The F - relation* L - fuzzy subalgebra *U* (*x, y*) *is open in* (*X, Y, T* ) *.*

*V*(*x, y*) *is any F - relation L - fuzzy subalgebra contained in U* (*x, y*)*. Since U* (*x, y*) *is* open, and *V*(*x, y*) *⊂ U* (*x, y*)*, V*(*x, y*) *⊂ U* (*x, y*) *⊂ U* (*x, y*)

∴ *U* (*x, y*) *is F - relation L - fuzzy neighbourhood of V*(*x, y*)*.*

*Conversely , for each F - relation L - fuzzy subalgebra V*(*x, y*) *contained in U* (*x, y*)*,*

*U* (*x, y*) *is L - fuzzy neighbourhood of V*(*x, y*)*.*

*for U* (*x, y*) *⊂ U* (*x, y*) *, by our assumption, U* (*x, y*) *is F - relation L - fuzzy neighbourhood* of *U* (*x, y*)*.*

*Hence there exits an open F - relation L - fuzzy subalgebra O*(*x, y*) *such that U* (*x, y*) *⊂* O(*x, y*) *⊂ U* (*x, y*)

*Hence U* (*x, y*) = *O*(*x, y*) *and U* (*x, y*) *is open in* (*X, Y, T* ) *.*

# Theorem 3.9.

*X, Y are TM-Algebras. R*1(*x, y*)*, R*2(*x, y*) *are the fuzzy relations of X, Y. AL - structrue* on F - relation L - Fuzzy Topological System (*X, Y, T* )*. µ*(*x, y*) *is F - relation L - fuzzy* subalgebra in (*X, Y, T* )*.* U(*x, y*) *is F - relation L - fuzzy neighbourhood system of F -* relation L - fuzzy subalgebra *µ*(*x, y*)*. then*

1. *The finite intersections of F - relation L - fuzzy subalgebras of* U(*x, y*) *belong to*

U(*x, y*)

1. *F - relation L - fuzzy subalgebra of* (*X, Y, T* ) *which contain a F - relation L - fuzzy subalgebra of* U(*x, y*) *belong to* U(*x, y*)

*Proof:*

1. *AL - structrue on F - relation L - Fuzzy Topological System* (*X, Y, T* )*.*

*µ*(*x, y*) *is F - relation L - fuzzy subalgebra in* (*X, Y, T* )*.* U(*x, y*) *is F - relation L*

*- fuzzy neighbourhood system of µ*(*x, y*)*.*

*The F- relation L - fuzzy subalgebras g*(*x, y*)*, h*(*x, y*) *∈* U(*x, y*)*. Hence g*(*x, y*) *and*

*h*(*x, y*) *are F - relation L -fuzzy neighbourhood of µ*(*x, y*)*.*

*Thus there exits open F - relation L - fuzzy subalgebras g*0(*x, y*) *and h*0(*x, y*) *Such* that *µ*(*x, y*) *⊂ g*0(*x, y*) *⊂ g*(*x, y*) *and µ*(*x, y*) *⊂ h*0(*x, y*) *⊂ h*(*x, y*) *respectively.*

*Hence µ*(*x, y*) *⊂ g*0(*x, y*) *∩ h*0(*x, y*) *⊂ g*(*x, y*) *∩ h*(*x, y*)

*⇒ g*(*x, y*) *∩ h*(*x, y*) *is F - relation L -fuzzy neighbourhood of µ*(*x, y*)*.*

*Hence the intersection of two F - relation L - fuzzy subalgebras of* U(*x, y*) *is again* a F- relation L - fuzzy subalgebra of U(*x, y*)

*Hence the intersection of any finite number of F- relation L - fuzzy subalgebras of*

U(*x, y*) *is again a F - relation L - fuzzy subalgebra of* U(*x, y*)

1. *g*(*x, y*) *is F - relation L - fuzzy subalgebra that contains a F- relation L - fuzzy subalgebra of* U(*x, y*) *say u*(*x, y*)*.*

*Hence g*(*x, y*) *contains a F- relation L - fuzzy neighbourhood u*(*x, y*) *of µ*(*x, y*)*.*

*That is u*(*x, y*) *⊂ g*(*x, y*) *, u*(*x, y*) *∈* U(*x, y*)

*since u*(*x, y*) *is a F - relation L - fuzzy neighbourhood of µ*(*x, y*) *then by definition* there exists a open F- relation L - fuzzy subalgebra *o*(*x, y*)*.*

*⇒ µ*(*x, y*) *⊂ o*(*x, y*) *⊂ u*(*x, y*) *⊂ g*(*x, y*)*.*

*Therefore µ*(*x, y*) *⊂ o*(*x, y*) *⊂ g*(*x, y*)

*⇒ g*(*x, y*) *is F - relation L- fuzzy neighbourhood of µ*(*x, y*)*.*

∴ *g*(*x, y*) *∈* U(*x, y*)

# Theorem 3.10.

*X, Y are TM-Algebras. R*1(*x, y*)*, R*2(*x, y*) *are the fuzzy relations of X, Y. AL - structrue* on F - relation L - Fuzzy Topological System (*X, Y, T* )*. µ*(*x, y*) *is a F - relation L - fuzzy* subalgebra in (*X, Y, T* )*.*

1. *ν◦*(*x, y*) *is open in* (*X, Y, T* ) *and is the largest open F - relation L - fuzzy subalgebra contained in µ*(*x, y*)*.*
2. *F - relation L - fuzzy subalgebra µ*(*x, y*) *is open in* (*X, Y, T* ) *if and only if*

*µ*(*x, y*) = *ν◦*(*x, y*)

*Proof:*

1. *AL - structrue on F - relation L - Fuzzy Topological System* (*X, Y, T* )*. µ*(*x, y*) *is F - relation L - fuzzy subalgebra in* (*X, Y, T* )*.*

*Then by definition of F - relation L - fuzzy interior , σ◦*(*x, y*) *is again F - relation* interior L - subalgebra of *µ*(*x, y*)*.*

*Hence there exist an T- open F - relation L - fuzzy subalgebra o*(*x, y*) *such that*

*ν◦*(*x, y*) *⊂ o*(*x, y*) *⊂ µ*(*x, y*)*.*

*But o*(*x, y*) *is F - relation L - fuzzy interior L - fuzzy subalgebra of*

*µ*(*x, y*)*, o*(*x, y*) *⊂ ν◦*(*x, y*) *Hence ν◦*(*x, y*) = *o*(*x, y*)*.*

*Thus ν◦*(*x, y*) *is open in* (*X, Y, T* ) *and is the largest open F - relation L - fuzzy* subalgebra contained in *µ*(*x, y*)*.*

1. *Suppose F - relation L - fuzzy subalgebra µ*(*x, y*) *is open.*

*If µ*(*x, y*) *is open , then µ*(*x, y*) *⊂ ν◦*(*x, y*) *and ν◦*(*x, y*) *is F - relation L - fuzzy* interior of *µ*(*x, y*)*.*

*Hence µ*(*x, y*) = *ν◦*(*x, y*)

*Conversely , Suppose µ*(*x, y*) = *ν◦*(*x, y*)

*Hence by definition of F - relation L- fuzzy interior, the union of all F - relation* L- fuzzy interior of *µ*(*x, y*) *is called the interior of µ*(*x, y*) *and is denoted by ν◦*(*x, y*)*.*

∴ *µ*(*x, y*) *is a F - relation L - fuzzy neighbourhood of ν◦*(*x, y*)*.*

*Therefore F - relation L - fuzzy subalgebra µ*(*x, y*) *is open in* (*X, Y, T* ) *.*

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