A Portrayal of Integer Solutions to Non-homogeneous

Ternary Cubic Diophantine Equation



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Abstract

This chapter is concerned with the problem of determining varieties of

non-zero distinct integer solutions to the non-homogeneous ternary cubic

diophantine equation 

Different sets of integer solutions to the above equation are obtained by reducing

it to the equation ,which is solvable , through employing suitable transformations.

Keywords : ternary cubic , non-homogeneous cubic ,integer solutions

Introduction

One of the interesting areas of Number Theory is the subject of diophantine equations which has fascinated and motivated both Amateurs and Mathematicians alike. It is well-known that diophantine equation is a polynomial equation in two or more unknowns requiring only integer solutions. It is quite obvious that Diophantine equations are rich in variety playing a significant role in the development of Mathematics .The theory of Diophantine equations is popular in recent years providing a fertile ground for both Professionals and Amateurs. In addition to known results , this abounds with unsolved problems. Although many of its results can be stated in simple and elegant terms , their proofs are sometimes long and complicated. There is no well unified body of knowledge concerning general methods .A Diophantine problem is considered as solved if a method is available to decide whether the problem is solvable or not and in case of its solvability , to exhibit all integers satisfying the requirements set forth in the problem. The successful completion of exhibiting all integers satisfying the requirements set forth in the problem add to further progress of Number Theory as they offer good applications in the field of Graph theory ,Modular theory ,Coding and Cryptography ,Engineering ,Music and so on. Integers have repeatedly played a crucial role in the evolution of the Natural Sciences. The theory of integers provide answers to real world problems.

It is well-known that diophantine equations , homogeneous or non-homogeneous ,have aroused the interest of many mathematicians.. It is worth to observe that Cubic Diophantine equations fall in to the theory of Elliptic curves which are used in Cryptography. In particular , one may refer [1-10] for cubic equations with three and four unknowns.

The main thrust of this Chapter is to exhibit different sets of integer solutions to an interesting ternary non-homogeneous cubic equation given by  by using elementary algebraic methods. The outstanding results in this study of Diophantine equation will be useful for all readers.

Method of analysis

The non-homogeneous ternary cubic diophantine equation to be solved

is given by

 (1)

Introduction of the linear transformations

 (2)

in (1) leads to

 (3)

The process of obtaining different sets of integer solutions to (1) is illustrated below :

Illustration 1

It is observed that (3) is satisfied by

 (4)

and

 (5)

Using (4) in (2) , we get

 (6)

Thus , (5) & (6) represent the integer solutions to (1).

Illustration 2

Taking

 (7)

in (3) leads to

 which is satisfied by

 (8)

and

 (9)

In view of (7) , note that

 (10)

Using (8) & (10) in (2) , we get

 (11)

Thus , (9) & (11) represent the integer solutions to (1).

Illustration 3

Taking

 (12)

in (3) leads to

 which is satisfied by

 (13)

and

 (14)

In view of (12) , note that

 (15)

Using (13) & (15) in (2) , we get

 (16)

Thus , (14) & (16) represent the integer solutions to (1).

Illustration 4

Taking

 (17)

in (3) , it is written as

 (18)

It is possible to choose the values of  so that the R.H.S. of (18) is a

perfect square and hence the corresponding values of are obtained.

In view of (17) ,the values of are found .Substituting these values of 

in (2) , the respective integer solutions to (1) are found. The above process is

exhibited below:

Let

 (19)

which is satisfied by



Assume

 (20)

to be the second solution to (19). Substituting (20) in (19) and simplifying,

we have



In view of (20) , one has



The repetition of the above process leads to the general solution to (19) as



 (21)

From (18) , it is seen that



Also , from (17) ,note that



In view of (2) ,the integer solutions to (1) are given by



along with (21) .

Illustration 5

Taking

 (22)

in (3) , it is written as

 (23)

Let

 (24)

which is satisfied by



Assume

 (25)

to be the second solution to (24). Substituting (25) in (24) and simplifying,

we have



In view of (25) , one has



The repetition of the above process leads to the general solution to (24) as



 (26)

From (23) , it is seen that



Also , from (22) ,note that



In view of (2) ,the integer solutions to (1) are given by



along with (26) .

Illustration 6

Assume

 (27)

Express the integer  on the R.H.S. of (3) as the product of complex

Conjugates as follows

 (28)

Substituting (27) & (28) in (3) and employing the method of factorization ,

consider

 (29)

Equating the real and imaginary parts in (29) , the values of are found.

In view of (2) ,the values of are given by

 (30)

Thus , (27) & (30) give the integer solutions to (1).

Note 1:

Apart from (28) , one may consider the integer  on the R.H.S. of (3) as



giving a different set of integer solutions to (1).

Conclusion:

In this paper, we have made an attempt to find infinitely many non-zero distinct integer solutions to the non- homogeneous cubic equation with three unknowns given by . To conclude, one may search for other choices of solutions to the considered cubic equation with three unknowns and higher degree diophantine equations with multiple variables.

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