**A Distributed Time Delay model with a Single Prey and Two predators**

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**ABSTRACT:**

 **I**n this chapter, we propose a mathematical model comprising a single prey and two predators. In this model two predators (x2, x3) are preying on common prey (x1) and are neutral to each other. All the three species have limited their own natural resources. A Distributed type of delay is included in the interaction prey (x1) and second predator (x3). The system is described by system of integro differential equations. The co-existing state is identified and characterizes the local, global stability analysis at this state. The effect of Time delay on the dynamical behaviour of the system is studied using Numerical simulation**:**

**Keywords: Prey, Predator, stability analysis, Numerical simulation.**

**Mathematics Subject Classification:**34DXX

**I.INTRODUCTION**

The relation between prey-predator models is significant in biological relationships. Differential equations play a significant role to establish such relations. The application approach to study the dynamics of the models are by Braun [9] and Simon’s [10]. Mathematical modelling approach using differential equation was initiated by Lokta [1] and Volterra [2]. Stability analysis of biological, ecological, epidemical models is briefly discussed by Kapur [3, 4]. May R M [5], Murry [6] and Freed man [7] explained the wide range of ecological models with detailed analysis.

Naturally any biological or ecological phenomenon not only depend on the current values, but also dependent on previous history. The concept of time delay is proposed and introduced to study system dynamics which depend on previous history. The time lags are classified as discrete, continuous, and distributed type. The appropriate time lags for ecological systems are distributed type and are dealt by authors [11-12]. The time delays are influence the dynamics of the system and tend to destabilize or stabilizes the system. The systems with delay arguments and the qualitative analysis are widely studied by the authors [13-15]. These lags will change the stable equilibrium to unstable or vice versa. The time lags are also significant in epidemiology and the instability tendencies in HIV and SIR epidemic models are dealt by Karuna [16] and Ranjith [17. Three species prey, predator and super predator models were dealt by Shiva Reddy [18]. The three species distributed type delay models with different iterations were extensively studied by Paparao [19-24]. These delay kernels play switchover behaviour from stable to unstable vice versa. In spite of above models, we take a single prey and two predators’ model for investigation. We studied the dynamics of the model at co-existing state and prove that the system is both locally and globally asymptotically stable.

**II. Formation of Mathematical Model**

The basic model is with single prey and two predators preying on the same prey species was dealt by Shiva Reddy [18] with exponential growth model. The system dynamics was studied at all possible equilibrium points and shown that the system is both locally and globally asymptotically stable. We proposed the mathematical model with logistic grow type of single prey and two predators. The two predators are generalist type. Paparao et al [25] studied the dynamics of the model and shown that the system is asymptotically stable globally. In spite of that we infuse a distributed time delay in prey-predator model (logistic) in the interaction prey and second predator. The model is characterized by the system of integro differential equations given by

 (2.1)

With the following notations

x1(t) Prey population, x2(t) First predator population, x3(t) second predator population

: Growth rates of three populations

: Mutual interference strengths of three species

Li Carrying capacities of three species; are weight kernels.

Let and substitute in equation (1.1) becomes

 (2.2)

Assume the solutions for the above model (2.2) as

 we get

 (2.3)

All the constants are assumed to be positive

From equation (2.3) we can write

 (2.4)

From the equation (2.4) **it** is evident that

 (2.5)

From the above equation (2.5) the system (2.1) admits positive solutions in .

**III. Existence of Equilibrium:**

The co-existing state of the system (2.1) exist if the following conditions holds good.

1. (ii)

The co-existing state given by

 (3.1)

**IV. Local Stability Analysis**

**Theorem 4.1:** The system (2.3) is locally asymptotically stable at co-existing state

**Proof:** Consider the Jacobean matrix for the system (2.3) is

(4.1.1)

With the characteristic equation

Where

Calculate the following determinates

 Clearly the product is also positive.

Hence

Since all three determinates are positive, by Routh -Hurwitz criteria the co-existing state

**V. Global Stability**

**Theorem 5.1:** The co-existing state is globally asymptotically stable

**Proof:** consider the suitable Lyapunov’s function given below.

 (5.1.1)

Clearly

The time derivate of V along the solutions of equations (2.1) is

 (5.1.2)

 (5.1.3)

Choose the proper set of values for Then (5.1.3) becomes

choose m2 = 1

Hence

Therefore, the co –existing state is globally asymptotically stable

**Theorem 5.2:** The system (1.3) cannot have any periodic orbits in the interior of the quadrant.

**Proof:** Using Bendixen -Dulac criterion we establish a dulac function

And define

 (5.2.1)

Clearly function is a positive (since the population strengths are positive values) in the interior positive octant of space.

Calculate which is given by

 *=*

*=*

This shows that

Therefore does not change the sign and identically zero in the positive quadrant of space. hence the system (1.3) does not produce any closed orbits and periodic oscillation.

**VI. Numerical Simulation:**

Numerical simulation is performed using the parametric values given in example (5.1) exponential kernel described with becomes

 (6.1)

The figure(A) represents Time series plot and figure (B) represents phase portrait

Example:5.1

with the above parametric values, the simulation is carried out for the system of equations (6.1) without impose delay arguments converging to fixed equilibrium point E (45,374,234) shown in the graphs 6.1(A) & 6.1 (B) respectively

 

 **Fig. 6.1(A) Fig. 6.1 (B)**

Defined as follows with different kernel strengths as

 **Case (1) for a = 0.01, λ = 5.**

 

 **Fig. 6.1.1(A) Fig. 6.1.2 (B)**

 Converging to fixed equilibrium point E (81,554,150)

**Case (ii) a = 0.1, λ = 5** for this kernel strengths the system (2.3) converges to a fixed equilibrium point E (80,551,152)

**Case (iii) a = 0.5, λ = 5** for this kernel strengths the system (2.3) converges to a fixed equilibrium point E (78,542,163)

 **Case (iv) a = 5, λ = 5** for this kernel strengths the system (2.3) converges to a fixed equilibrium point E (65,472,211)

**Case(v) a = 50, λ = 50** for this kernel strengths the system (2.3) converges to a fixed equilibrium point E (65,472,211)

As on the weight kernel strength increase from 0.01 < a< 50 and 0.5 < λ < 50 the prey population & first predator population decreases and second predator population increase when compared with the dynamics of the system without delay arguments.

**VI. Conclusion:**

In this chapter we studied the delay dynamics of three species ecological model with a single prey and two predators. The system of integro-differential equations describes the system. The co-existing state is identified and studied the local and global dynamics of the model. The system does not admit any periodic oscillations shown using Dulac criteria. Hence the system is both locally and globally asymptotically stable.

Numerical simulation is performed with suitable parametric values and exponential type delay kernel and shown that the system is stable for different types of delay kernel strengths and the weight are significant in influencing the population dynamics.

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