**Applications of Soham Transform for Solving Volterra Integral Equation of Second Kind**

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**ABSTRACT**

 Many advanced scientific and engineering problems can be theoretically represented as linear Volterra integral equations. In this work, soham transform is applied and used to find the solution of Volterra integral equations of 2nd kind. To demonstrate the applicability of the soham transform, some numerical problems were presented and solved by the sohamtransform. Numerical results show that the soham transform is very effective for obtaining the accurate solution of the 2nd kind Linear Volterra integral equations.

**Keywords:** Volterra integral equations of 2nd kind, soham transform, soham inverse transform, Convolution theorem.

1. **INTRODUCTION :** The 2nd kind of linear Volterra integral equation is defined as follows

Here the function f(y) and kernel k(y, t) are known real- valued functions. The unknown Function

is denoted by and is a non-zero real parameter.

We use the soham transform to solve linear Volterra integral equations of 2nd kind. Recently, integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. It is very useful and effective technique for solving a linear differential equations and system of such equations and integral equations under the given initial conditions. To provide exact solution of problem without lengthy calculations is the important feature of integral transforms.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [1] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [2] introduced Soham transform in November 2021. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Patil et al [5, 6, 7] used Kushare transform solving different problems.

Patil with Tile and Shinde [4] used Anuj transform and solved Volterra integral equations for first kind. Suryawanshi et al [3, 8] used Soham transform for solving volterra integral equations and mathematical models occurring in health science and biotechnology.

This paper is organized as follows. Introduction is in first section. Second section is for preliminary concepts. Third section is devoted to state and prove Convolution theorem of Soham transform. Fourth section is reserved for applications of Soham transform for solving Volterra integral equations.

1. **PRELIMINARIES:** In this section we state some basic requirements. Now we state some required definitions, properties and formulae.
	1. **Soham transform:** Soham Transform denoted by the operator S(.) is defined by the integral equation dt

 is non zero real numbers t ≥ 0,

* 1. **Inverse Soham Transform:** Inverse Soham transform is denoted as follows:

If Soham transform of f(t) is P(v) then inverse Soham transform is defined as

* 1. **Properties of Soham Transform:**

 In this section we state some properties of soham transform which are useful for us [ 1].

1. **Linearity property**: If 𝑓1 (t) and 𝑓2(t) be two functions of t andc1 and c2 be any two constants then

2. Transform of derivative: Let P(v) Soham transform of f(t) i.e . [S[f(t)] = P(v)] then

#  Soham Transform of Elementary Functions:

The sufficient conditions for the existence of Soham transform are that for t ≥ 0 the function f(t) be piecewise continuous and of exponential order, otherwise Soham transform may or may not exist. In this section we state Soham transform of some elementary functions.

Table: Soham transform of some functions

|  |  |  |
| --- | --- | --- |
| Sr.No. | f(t) |  |
| 1 | 1 |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |

1. **CONVOLUTION THEOREM:**

In this section we state and prove convolution theorem.

The convolution of two function f(t) and g(t) is. ,

then

**Proof:** Applying Soham transform.

Change order of integration and put

Thus convolution theorem for Soham transformations:

is proved.

1. **SOHAM TRANSFORM FOR CONVOLUTION TYPE 2ND KIND LINEAR VOLTERRA INTEGRAL EQUATIONS**

In this study, the kernel will be assumed to be a difference kernel, as described by the difference(y-t).The 2nd kind of Volterra integral equation can thus expressed as:

 …..(1)

The Soham integral transform method is applied to both sides of (1), yielding

…..(2)

Now using convolution theorem of Soham integral transform on equation (2), we have

Inverting the Soham integral transform on equation (3), We obtain:

Equation (4) is the required solution of equation (1)

**5. NUMERICAL APPLICATIONS**

Some applications are presented in this part to show the unity of Soham integral transform for solving Volterra integral equation of the 2nd kind. Now we solve following Volterra integral equations of second kind.

1. Consider the Volterra integral equation of the 2nd kind :

The soham transform is applied to both sides of equation (5), yielding,

Now using the Soham transform convolution theorem on equation (6), we have,

S{(y)}=+vs{siny}.s{(y)}

s{(y)}[1−] =

s{(y)}= ….. (7)

We obtained required solution (5) by applying the inverse Soham transformation of both sides of (7)

(y) = 1

It is required solution.

1. Take Volterra integral equation of the 2nd kind

=y+y-t)dt ….…(8)

Applying soham transform to both sides of equation (8), we get,

s{= s{y}+s{y-t)dt}

s{}=s{y}+s{siny\*} …….(9)

By convolution theorem of Soham transformation,

By applying inverse Soham transformation to both sides of equation (10),

(y)={ +{

(y)= t +

It is the required solution.

1. Now consider the Volterra integral equation of the 2nd kind:

Applying Soham transform on both sides,

By using convolution theorem of Soham transform,

S{(y)}=+ v.s{}. S{(y)}

+

Applying inverse Soham transform,

It is the required solution.

(D)Take the Volterra integral equation of 2nd kind:

 (y)=-cosy -2(t).dt ..……..(12)

Applying Soham transform on both sides, we get

Using convolution theorem,

We apply the inverse Soham transform to above equation.

It is the required solution.

1. Consider the Volterra integral equation of the 2nd kind:

Applying Soham transform, on both side we get,

By using convolution theorem of Soham transform

 S{(y)}=v s{1}.s{(g)}

By using inverse Soham transform

}

 is the required solution.

1. **CONCLUSION:** We applied Soham integral transform to solve the problems on Volterra integral transform successfully. Answers obtained are same as obtained by other methods.

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