**Applications of Soham Integral Transform toObtain Solution of Volterra Integral Equation of Second Kind**

1. P.Patil,

[sdinkarpatil95@gmail.com](mailto:sdinkarpatil95@gmail.com)

Department of Mathematics,

K. R. T. Arts, B. H. Commerce and A.M. Science College, Nashik.

**ABSTRACT**

Linear Volterra integral equations occurs in many advanced scientific and engineering fields . In this work, Soham transform is used to obtain the solution of Volterra integral equations of 2nd kind. To demonstrate the applicability of the Soham transform, some numerical problems were presented and solved by the Soham transform. Numerical results show that the Soham transform is very effective for obtaining the accurate solution of the 2nd kind Linear Volterra integral equations.

**Keywords:** Volterra integral equations of 2nd kind, Soham transform, Soham inverse transform, Convolution theorem.

1. **INTRODUCTION :** The 2nd kind of linear Volterra integral equation is defined as follows

Here the function f(y) and kernel k(y, t) are known real- valued functions. The unknown Function

is denoted by and is a non-zero real parameter.

We solve linear Volterra integral equations of 2nd kind by using the Soham transform To obtain the solutions of advanced problems occurring in many fields like Science, Engineering, Technology, Commerce and Economics, the most useful and simple mathematical technique is use of integral transform. It is very useful and effective technique for solving a linear differential equations and system of such equations and integral equations under the given initial conditions. The important feature of integral transforms is to provide exact solution of the problem without lengthy calculations.

As integral transforms has this important feature of obtaining solution of the problem using small number of calculations, many researchers are attracted to this field. They make themselves engaged in introducing various new integral transforms with various Kernels on various domains. Recently,Kushare transform is introduced by Kushare and Patil [1] and used it to solve differential equations in time domain. Further, in November 2021 new integral transform named as Soham transform is introduced by Savita Khakale and Dinkar Patil [2].Further, Patil and some authors [5, 6, 7] used Kushare transform for solving different problems.

Anuj integral transform is used by Patil et al [4] to solve the Volterra integral equations for first kind. Patil together with Suryawanshi [3, 8] solved volterra integral equations and mathematical models occurring in health science and biotechnology by using Soham transform.

We organize this paper as follows. First section is for introduction . Second section is reserved for preliminary concepts. In third section we state and prove Convolution theorem of Soham transform. Fourth section is reserved for applications of Soham transform for obtaining the solution of Volterra integral equations.

1. **PRELIMINARIES:** We state some basic required concepts in this section. Now we state some required definitions, properties and formulae.
   1. **Soham transform:** Soham Transform is denoted by the operator S(.) and it is defined by the integral equation dt

where is a non zero real number, t ≥ 0,

* 1. **Inverse Soham Transform:**

If Soham transform of the function f(t) is denoted by P(v) then inverse Soham transform is defined as

* 1. **Properties of Soham Transform:**

In this section, we state some useful properties of Soham integral transform [ 1].

1. **Linearity property**: If 𝑓1 (t) and 𝑓2(t) are two functions of t andc1 and c2are any two constants then

2. **Transform of derivative**: Let P(v) be the Soham transform of the function f(t) i.e . [S[f(t)] = P(v)] then

# Soham Transform of Elementary Functions:

“For t ≥ 0 the function f(t) is piecewise continuous and is of exponential order” , these are the sufficient conditions for the existence of Soham transform. If these conditions are not satisfied then Soham transform of that function may or may not exist.

Soham transform of some elementary functions are stated in following table.

Table: Soham transform of some functions

|  |  |  |
| --- | --- | --- |
| Sr.No. | f(t) |  |
| 1 | 1 |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |

CONVOLUTION THEOREM:

In this section we state and prove convolution theorem.

We define The convolution of two function f(t) and g(t) as. ,

Theorem: If f(t) and g(t) are two functions such that and ,

then

Proof: Applying Soham transform.

Change order of integration and put

Thus the convolution theorem for Soham transform:

is proved.

1. **SOHAM TRANSFORM FOR CONVOLUTION TYPE 2ND KIND LINEAR VOLTERRA INTEGRAL EQUATIONS**

In this study, the kernel will be assumed to be a difference kernel, as described by the difference(y-t).The 2nd kind of Volterra integral equation can thus expressed as:

…..(1)

The Soham integral transform method is applied to both sides of (1), yielding

…..(2)

Now we use convolution theorem of Soham integral transform on both sides of the equation (2), we have

Inverting the Soham integral transform on equation (3), We obtain:

Equation (4) is the required solution of equation (1)

**5. NUMERICAL APPLICATIONS**

Some applications are presented in this part to show the unity of Soham integral transform for solving Volterra integral equation of the 2nd kind. Now we solve following Volterra integral equations of second kind.

1. Consider the Volterra integral equation of the 2nd kind :

The Soham transform is applied to both sides of equation (5), yielding,

Now using the Soham transform convolution theorem on equation (6), we have,

S{(y)}=+vs{siny}.s{(y)}

s{(y)}[1−] =

s{(y)}= ….. (7)

We obtained required solution (5) by applying the inverse Soham transformation of both sides of (7)

(y) = 1

It is required solution.

1. Consider Volterra integral equation

=y+y-t)dt ….…(8)

We apply Soham transform on both sides of equation (8), and obttain,

s{= s{y}+s{y-t)dt}

s{}=s{y}+s{siny\*} …….(9)

By using convolution theorem of Soham transformation,

By applying inverse Soham transformation to both sides of equation (10),

(y)={ +{

(y)= t +

It is the required solution.

1. Now consider the Volterra integral equation of the 2nd kind:

Applying Soham transform on both sides,

By using convolution theorem of Soham transform,

S{(y)}=+ v.s{}. S{(y)}

+

Applying inverse Soham transform,

It is the required solution.

(D)Take the Volterra integral equation of 2nd kind:

(y)=-cosy -2(t).dt ..……..(12)

Applying Soham transform on both sides, we get

Using convolution theorem,

We apply the inverse Soham transform to above equation.

It is the required solution.

1. Consider the Volterra integral equation of the 2nd kind:

Applying Soham transform, on both side we get,

By using convolution theorem ofSoham transform

S{(y)}=v s{1}.s{(g)}

By using inverse Soham transform

}

is the required solution.

1. **CONCLUSION:** We applied Soham integral transform to solve the problems on Volterra integral transform successfully. Answers obtained are same as obtained by other methods.

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