**Numerical analysis of Darcy-Forchheimer flow and heat transfer over a stretching sheet with uniform heat source**

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**Abstract**

The present study investigates the Maxwell fluid's Darcy-Forchheimer flow and heat transfer across a uniform vertical extending heat source. The governing partial differential equations (PDEs) are transformed into non-linear ordinary differential equations (ODEs) using comparable procedures. The resultant ODEs are numerically solved using the shooting method and the Runge-Kutta fourth order approach. Through graphs and figures, the effects of important factors on velocity, temperature, skin friction coefficient, and local Nusselt number are depicted. The fluid velocity is discovered to be decreased by the local inertia parameter, which causes inertia drag, although a negative impact on the temperature field is shown.

**Keywords:** Darcy-Forchheimer model, heat transfer, stretching sheet, uniform heat source/sink.

1. **Introduction**

In many industrial processes, the flow issue in the boundary layer brought on by a continually moving or extending surface is significant. In the manufacturing process, continuous extrusion of the polymer from a die to a windup roller, which is situated a certain distance from a die, produces polymer sheets and filament. Through an ambient fluid or a fluid flowing at a certain defined velocity, the thin polymer sheet creates a constantly moving surface with a non-uniform velocity. Crane [1] investigated the boundary layer flow resulting from sheet stretching.

Due to its numerous practical applications in geothermal and oil reservoir engineering, as well as other geophysical and astrophysical investigations, the flow and heat transfer through a porous medium have attracted a lot of attention in recent years. The viscoelastic fluid across a stretched sheet has been researched by Nayak et al. Dessie and Kishan's [4] work on the heat transfer flow of a fluid over a stretched sheet included the incorporation of viscous dissipation and heat source/sink. The impact of changing viscosity and thermal conductivity on MHD heat and mass transmission over a stretched sheet has been examined by Swain et al. Viscous dissipation and chemical reactions on mixed convective Darcy-Forchheimer fluid flow have been taken into consideration by Mahdy and Chamkha [6]. These investigations have all been restricted to a porous media.

By taking into account the impact of thermal radiation, a new dimension is given to the study of flow and heat transmission in a viscous fluid across a stretched surface. In the manufacturing of polymers, the thermal radiation effect may be a key factor in managing the heat transfer process. The heat controlling parameters have a significant impact on the final product's quality. The understanding of radiative heat transport in the system may result in a product with the desired property. Engineering operations frequently take place at high temperatures, therefore understanding radiation heat transfer is crucial for designing the necessary machinery which include nuclear power plants, gas turbines, and different propulsion systems for airplanes, missiles, satellites, and spacecraft. The impact of heat radiation passing through a moving vertical porous plate has been studied by Makinde [7]. The impact of radiation on mixed convection stagnation point flow of nanofluid across a vertical stretched sheet has been investigated by Golafshan and Rahimi [8]. Swain et al. [9] conducted a slip flow study on MHD nanofluid across a vertical stretched sheet with higher order chemical reaction. Hayat et al. [10] studied the analysis of the heat and mass transport of a chemical reaction in a Darcy-Forchheimer flow. Bakar et al. [11] investigated the stagnation point flow in a porous medium with slip boundary conditions across a diminishing sheet. The Darcy-Forchheimer flow of Sisko nanomaterial with nonlinear heat radiation was numerically analysed by Uddin et al. [12]. MHD Darcy-Forchheimer nanofluid flow across a nonlinear stretching sheet was performed by Rasool et al. [13].

This study's goal is to construct a mathematical model of the heat transfer and Darcy-Forchheimer boundary layer flow across a vertically stretched sheet with non-uniform heat source/sink. Similarity transformations are used to convert the governing PDEs into non-linear ODEs, which are then, solved using the shooting approach and the Runge-Kutta fourth order method. Graphs are used to display and demonstrate the impacts of various factors in detail. Furthermore, tables have been used to calculate and thoroughly analyse the shearing stress and the rate of heat transfer at the plate.

1. **Mathematical Formulation**

In the presence of a homogeneous heat source and sink, we investigate a steady two-dimensional boundary layer flow and heat transfer over a vertically stretched sheet. The *x*-axis, which is selected to run parallel to the sheet and perpendicular to it, is supposed to be the direction of the flow. Let and  are the tangential and normal velocities of the fluid respectively. In the study of porous media flow analysis, the differential equations of fluid motion are based on Forchheimer, which takes into account the drag imposed by the porous medium. Under Boussinesq's approximation, the boundary layer equations for momentum and energy are as follows:



Fig. 1 Flow model and coordinate system

Continuity equation:

 (1)

Momentum equation:

 (2)

Equation of energy:

 (3)

The corresponding conditions are

 (4)

where are velocity components in *x* and *y* directions respectively, is the magnetic field strength, is the kinematic viscosity, is the electrical conductivity, is the density, is the thermal conductivity, is the temperature, is the ambient temperature of the fluid, is the specific heat, is the drag coefficient,  is the heat source/sink coefficient, is the permeability of the medium, is a constant, and is the heat transfer coefficient .

By using the following similarity transformations and non-dimensional variables 

the equations (1) – (4) can be written as

 (5)

 (6)

 (7)

where is the magnetic parameter, is the porosity parameter, is the local inertia parameter, is the Prandtl number, is the heat source/sink parameter, andis the Biot number.

The skin friction coefficient and local Nusselt number  are given by  and respectively.

Here, wall shear stress and wall heat flux where is the local Reynolds number.

1. **Results and Discussion**

The Runge-Kutta fourth order approach with shooting technique and MATLAB software are used to solve the coupled, non-linear PDEs (5) through (7). By contrasting the current study with that of Khan and Pop [14] as a specific case assignment,  as shown in Table 1, the validation has been achieved. The non-dimensional parameters and  have been fixed during discussion, unless otherwise specified.

**Table 1** Comparison of the values of 

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  | Khan and Pop [14] | Present study |
| 0.07 | 0.0663 | 0.064295 |
| 0.2 | 0.1691 | 0.168294 |
| 0.7 | 0.4539 | 0.451590 |
| 2 | 0.9113 | 0.910680 |
| 7 | 1.8954 | 1.894921 |
| 20 | 3.3539 | 3.353507 |
| 70 | 6.4621 | 6.461822 |



Fig. 2 Influence of on velocity and temperature profiles

 The influence of the magnetic parameter on the velocity and temperature profiles is seen in Fig. 2. A Lorentz force is created by an increase in the magnetic parameter and moves fluid in the opposite direction of the fluid flow. This force has a propensity to slow down the flow of fluid, which causes the fluid to generate greater heat. As a result, an increase in causes the fluid in the flow domain to move more slowly and warm up. Fig. 3 is depicted to study the influence of local inertia parameter on velocity and temperature profiles in presence and absence  of porous matrix. It is elucidated that on increasing inthe fluid velocity decreases. Forchheimer number provides for the inertia effects caused by porous media and pressure drop disturbed by fluid-solid interaction, which predominates the viscous interference, in cases of porous spaces with larger pore sizes. Thus, an increase in the local inertia parameter results in increased flow resistance, which reduces the fluid velocity while increasing the fluid temperature since more heat is produced by the porous medium.

 The effect of the heat source/sink parameter  on temperature distribution is examined in Fig. 4. This figure makes it clear that when the heat source/sink parameter rises, the thermal boundary layer thickness also increases.

 

Fig. 3 Influences of and on velocity and temperature profiles



Fig. 4 Influence of on temperature profile

**Table 2** Values of skin friction coefficientand Nusselt number 

when 

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *M* |  |  |  |  |  |
| 0.1 | 0.1 | 0.1 | 0.1 | 1.290355 | 0.087724 |
| 0.5 |  |  |  | 1.620293 | 0.095979 |
| 1.0 |  |  |  | 1.735964 | 0.096853 |
|  | 0.5 |  |  | 1.780571 | 0.094248 |
|  | 1.0 |  |  | 1.928377 | 0.093136 |
|  |  | 0.3 |  | 1.928348 | 0.272901 |
|  |  | 0.5 |  | 1.928347 | 0.367355 |
|  |  |  | 0.3 | 1.919284 | 0.343609 |
|  |  |  | 0.5 | 1.901410 | 0.308226 |

 Table 2 is computed to observe the impacts of important physical parameters and on skin friction coefficient and Nusselt number. The skin friction coefficient is getting enhanced on increasing values of and whereas slightly decreases with higher values of. On the other hand, local Nusselt number is getting enhanced on increasing values of and whereas decreases on increasing either of and. These results are well supported by Seth et al. [15]

1. **Concluding remarks**

The main findings of the present study are:

* Decreasing the fluid's velocity while increasing the thermal resistance, which raises the temperature profile, are effects of increasing the magnetic parameter values.
* Although the local inertia parameter (Forchheimer parameter), which causes inertia drag, decreases fluid velocity, a negative impact on the temperature field is seen.
* Magnetic parameter and local inertia parameter have the same impact on skin friction coefficient but opposing effects on local Nusselt number.
* Biot number and Heat source/sink parameter have an increasing influence on temperature distribution.

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