**SIMILARITY MEASURES OF FERMATEAN NEUTROSOPHIC SETS BASED ON THE COSINE FUNCTION AND THEIR APPLICATIONS**

1 Radha R, 2  Princy R , 3Gayathri P ,4Gomathi S , 5Kavitha A, 6Kalamani A

**ABSTRACT:**

In this study, we analyse the degree of hesitation, non-membership, and membership in Fermatean Neutrosophic sets (FNSs) and give similarity metrics between them based on the cosine function. Next, we utilise these similarity measures along with weighted measures amongst FNSs for diagnosing medical conditions and identifying patterns. Lastly, two instances are provided to show how effective similarity measures are at identifying patterns and making medical diagnoses.

1. **SOME SIMILARITY MEASURE BASED ON THE COSINE FUNCTION FOR FERMATEAN NEUTROSOPHIC SETS**

**3.1 Cosine Similarity Measure for FNSs**

Let be a FNS in an universe of discourse , the FNS is characterized by the degree of membership , the degree of non-membership , and the degree of hesitation , , which can be considered as a vector representation with the three elements. Therefore, a cosine similarity measure and a weighted cosine similarity measure for FNSs are proposed in an analogous manner to the cosine similarity measure based on Bhattacharya’s distance [21, 6]and cosine similarity measure for IFS [28]**.**

Suppose that there are two FNSs and in the universe of discourse , we further propose the cosine similarity measures between FNSs as follows:

------------(17)

If we take n=1, then the cosine similarity measure between FNSs and becomes the correlation coefficient between FNSs and . Therefore, the cosine similarity measure between and also satisfies the following properties:

1. .
2. if

Proof:

1. It is obvious that the proposition is true according to the cosine value.
2. It is obvious that the proposition is true.
3. When there are , and So there is .

Therefore, we have finished the proofs.

If we consider the weights of , a weighted cosine similarity measure between FNSs and is proposed as follows:

--------------(18)

Where is the weight vector of , with , In particular, if , then the weighted cosine similarity measure reduces to cosine similarity measure. That is to say, if we take then there is . Obviously, the weighted cosine similarity measure of two FNSs and also satisfies the following properties:

1. .
2. if .

Similar to the previous proof method, we can prove the above three properties.

In the following, we shall investigate the distance measure of the angle as

It satisfies the following properties:

1. if ;
2. , if
3. if
4. if for any FNS .

Proof:

Obviously, satisfies the property (1) - (3). In the following, will be proved to satisfy the property (4).

For any , , Since Equation (16) is the sum of terms, let us investigate the distance measures of the angle between the vectors:

and

where

For three vectors ,

in one plane, if Then, it is obvious that according to the triangle inequality. Combining the inequality with Equation (16), we can obtain Thus satisfies the property (4). So we finished the proof.

* 1. **Similarity measures of FNSs based on cosine function:**

Based on the cosine function, in this section, we shall propose two cosine similarity measures between FNSs and analyse their properties.

**Definition 3.2.1:**

Suppose that there are two FNSs and in the universe of discourse , we further propose the cosine similarity measures between FNSs as follows:

--------------(19)

--------------(20)

Where the symbol is the maximum operator.

**Proposition 3.2.2:**

For any two FNSs and in the cosine similarity measures should satisfy the following properties (1) – (4):

2. If is a FNS in and , then and .

Proof:

1. Since the value of the cosine function is within [0,1], the similarity measure based on the cosine function is also within [0,1]. Thus, there is .
2. For any two FNSs and in if , then and for Thus,

So,

If this implies for Since cos(0) =1. Then, there are

and for Hence

.

1. Proof is straightforward.
2. If then there are , and for Then, , and

Thus, we have

,

,

,

,

and

.

Hence, and for as the cosine function is a decreasing function with the interval

Thus, the proofs of these properties are completed.

In many situations, the weight of the elements should be taken into account. For example, in Multiple Attribute Decision Making (MADM), the considered attributes usually have different importance, and thus need to be assigned different weights. As a result, two weighted cosine similarity measure between FNSs and is proposed as follows:

--------------(21)

--------------(22)

Where is the weight vector of , with , and the symbol is the maximum operator. In particular, if , then the weighted cosine similarity measure reduces to cosine similarity measure. That is to say, if we take then there is .

Obviously, the weighted cosine similarity measures also satisfy the axiomatic requirements of similarity measures in Proposition 2.

**Proposition 3.2.3:**

For any two FNSs and in the cosine similarity measures should satisfy the following properties (1) – (4):

3. If is a FNS in and , then W and .

By using similar proof in Proposition 1, we can give the proofs of these properties (1) – (4).

**3.3 Similarity Measures of FNSs based on the Cotangent Function:**

In this section, we shall propose two cotangent similarity measures between FNSs.

**Definition 3.3.1**:

Suppose that there are two FNSs and in the universe of discourse , we further propose the cotangent similarity measures between FNSs as follows:

--------------(23)

--------------(24)

Where the symbol is the maximum operator.

**Proposition 3.3.2:**

For any two FNSs and in the cotangent similarity measures should satisfy the following properties (1) – (4):

1. If is a FNS in and , then and .

**Proof:**

1. Since,

,

It is obvious that the cotangent function are within 0 and 1.

1. It is obvious that the proposition is true.
2. When , then obviously . On the other hand if then,

and for

This implies

1. If then we can write , and for Then, , and

The cotangent function is decreasing function within the interval .

Hence we can write and .

Thus, the proofs of these properties are completed.

In many situations, the weight of the elements should be taken into account. For example, in Multiple Attribute Decision Making (MADM), the considered attributes usually have different importance, and thus need to be assigned different weights. As a result, two weighted cotangent similarity measure between FNSs and is proposed as follows:

--------------(25)

----------------(26)

Where is the weight vector of , with , and the symbol is the maximum operator. In particular, if

, then the weighted cotangent similarity measure reduces to cotangent similarity measure. That is to say, if we take then there is .

**Proposition 3.3.3:**

For any two FNSs and in the cosine similarity measures should satisfy the following properties (1) – (4):

3. If is a FNS in and , then W and .

By using similar proof in Proposition 3, we can give the proofs of these properties (1) – (4)

4.**APPLICATIONS**

In this section, the cosine and cotangent similarity measures for FNSs are applied to pattern recognition and medical diagnosis to illustrate the feasibility of the proposed methods and deliver a comparative analysis

* 1. **Example 1: Pattern Recognition**

Let us consider, a three known patterns which are represented by the FNSs: in the feature space as

Consider an unknown pattern that will be recognized, where

The purpose of this problem is classify the pattern in one classes and . For it, the proposed similarities degrees have been computed from to and are given in Table 1.

|  |  |  |  |
| --- | --- | --- | --- |
| Similarity Measures |  |  |  |
|  | 0.8704 | 0.8320 | **0.8992** |
|  | 0.8747 | 0.8360 | **0.9041** |
|  | 0.9104 | 0.8863 | **0.9328** |
|  | 0.5967 | 0.5533 | **0.654** |
|  | 0.7695 | 0.7327 | **0.7898** |

Table 1: The similarity measures between and

From the numerical results presented in Table 1, we know that the degree of similarity between and is the largest one as derived by five similarity measures. That is, all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs. Compared with Garg’s correlation coefficient method [10], we can get the same result that all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs.

If we consider the weight of are 0.5, 0.3 and 0.2 respectively. Then we use the proposed weighted similarities measures have been computed from to and are given in Table 2.

|  |  |  |  |
| --- | --- | --- | --- |
| Similarity Measures |  |  |  |
|  | 0.8244 | 0.8145 | **0.8692** |
|  | 0.8631 | 0.8622 | **0.8808** |
|  | 0.8938 | 0.8975 | **0.9221** |
|  | 0.5818 | 0.5877 | **0.6161** |
|  | 0.7503 | 0.7426 | **0.7753** |

Table 2: The weighted similarity measures between and

From the numerical results presented in Table 2, we know that the weighted similarity measures between and is the largest one as derived by five similarity measures. That is, all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs. Compared with Garg’s correlation coefficient method [10], we can get the same result that all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs.

* 1. **Example 2: Medical Diagnosis**

Let us consider a set of diagnosis

and a set of symptoms

Suppose that a patient, with respect to all symptoms, can be depicted by the following FNS:

( (

And then each diagnoses can be viewed as FNSs with respect to all the symptoms as follows:

(

The purpose of this problem is classify the pattern in one classes . For this, the proposed similarities measures have been computed from to and are given in Table 3.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Similarity Measures |  |  |  |  |  |
|  | 0.5811 | 0.8863 | 0.9288 | 0.9420 | **0.9469** |
|  | 0.7440 | 0.8752 | 0.8911 | **0.9208** | 0.8685 |
|  | 0.7644 | 0.9236 | 0.9200 | **0.9355** | 0.9224 |
|  | 0.5083 | 0.6002 | 0.6328 | **0.7018** | 0.6005 |
|  | 0.6628 | 0.7741 | 0.7700 | **0.8155** | 0.7717 |

Table 3: The similarity measures between and

From the numerical results presented in Table 3, expect for the , we know that the similarity measures between and is the largest one as derived by five similarity measures. That is, the four similarity measures assign the unknown class to the known class according to the principle of the maximum degree of similarity between FNSs. Compared with Garg’s correlation coefficients method [10] we can get same result that the four similarity measures assign the unknown class to the known class according to the principle of the maximum degree of similarity between FNSs expect for the

If we consider the weight of is respectively. Then we apply the proposed weighted similarities measures, which have been computed from to and are given in Table 4.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Similarity Measures |  |  |  |  |  |
|  | 0.5608 | 0.7994 | 0.8251 | 0.8235 | **0.8517** |
|  | 0.6889 | 0.7881 | 0.8020 | **0.8204** | 0.7699 |
|  | 0.7083 | 0.8280 | 0.8201 | **0.8382** | 0.8206 |
|  | 0.4830 | 0.5465 | 0.5780 | **0.6245** | 0.5304 |
|  | 0.6149 | 0.6986 | 0.6892 | **0.7327** | 0.6865 |

Table 4: The weighted similarity measures between and

From the numerical results presented in table 4, we get the following results:

1. For similarity measures , the degree of similarity between and P is the largest one, so the pattern P should belong to the class of known diagnoses according to the principle of the maximum degree of similarity between FNSs.
2. For similarity measures the degree of similarity between and P is the largest one, so the pattern P should belong to the class of known diagnoses according to the principle of the maximum degree of similarity between FNSs. At the same time, for this case compared with Garg’s correlation coeeficients method [10], we can get the same result that the pattern P should belong to the class of the known diagnoses according to the principle of the maximum degree of similarity between FNSs.