**Magnetohydrodynamic flow and heat transfer of Casson fluid over a stretching sheet in non-Darcy porous medium**

Arati Mishra1,\*, Kharabela Swain2

1,\*Principal, Vikash Residential School, Bhawanipatna, Odisha-766002, India

1,\*E-mail: mishra.aarti123@gmail.com

Department of Mathematics, Gandhi Institute For Technology, Bhubaneswar, Odisha-752054, India

2E-mail: kharabelaswain@gift.edu.in

**Abstract**

The current study examines how the Lorentz force affects heat transfer and Casson fluid flow across a stretched sheet immersed in a non-Darcy porous medium with slip boundary condition. Using appropriate similarity transformations, the governing partial differential equations (PDEs) are transformed into ordinary differential equations (ODEs), which are then numerically solved using the shooting approach of the Runge-Kutta method. Graphs and tables are used to show how several characteristics, including magnetic, porosity, inertia, and slip parameters, affect velocity, temperature, skin friction coefficient, and the local Nusselt number. It is found that Casson parameter supports raising the fluid temperature while lowering the velocity profile and the slip parameter enhances skin friction coefficient while reducing rate of heat transfer at sheet surface.

**Keywords** MHD,Casson fluid, slip boundary condition, Darcy-Forchheimer model.

**1. Introduction**

Numerous industrial and technological uses of the boundary layer flow across a stretching/shrinking sheet include cooling of electronic devices, issues with blood flow, the extraction of polymer sheets, wire coating, polymer processing, etc. First, the boundary layer flow past a stretched sheet was explored by Sakiadis [1]. A precise solution to the MHD flow of a viscous fluid across a stretched sheet was discovered by Fang et al. [2]. The slip effect over a stretched sheet with non-linear boundary conditions was investigated by Mahantesh et al. [3]. The MHD flow and heat transmission through a permeable stretched sheet under slip circumstances were investigated by Hayat et al. [4]. The effects of slip boundary conditions on the MHD boundary layer flow and heat transfer of a nanofluid via a permeable surface were investigated by Ibrahim and Shankar [5]. The MHD flow of a variable viscosity nanofluid across a radially expanding convective surface was examined by Makinde et al. [6]. By taking into account varying fluid characteristics, Swain et al. [7–10] greatly enriched the literature on stretching sheets. The effects of natural convection on the gliding motion of bacteria on a power-lawnanoslime across a non-Darcy porous medium were examined by Abou-Zeid et al. [11].

 Casson fluid, which simulates blood flow in constricted arteries, is a non-Newtonian fluid with yield stress. Blood exhibits Newtonian characteristics when it flows at high shear rates from bigger diameter arteries. But when it passes via narrow arteries with modest shear rates, it behaves in a non-Newtonian manner. In his research on the flow of blood, Casson [12] examined the Casson fluid model's applicability and discovered that the yield stress for blood is nonzero at low shear rates. Ibrahim et al. [13] numerically studied the influence of heat source and chemical reaction on the Casson nanofluid's MHD stagnation point flow across a nonlinear stretching sheet with slip boundary conditions. The analytical solution on MHD boundary layer flow of Casson fluid across a stretching/shrinking sheet with wall mass transfer was investigated by Bhattacharyya et al. [14]. Mukhopadhyay [15] investigated the heat transfer and Casson fluid flow across a nonlinearly extending sheet. Recently, numerical analysis of the three-dimensional MHD Casson nanofluid flow across a stretched sheet is reported in [16].

 Analysis of the impact of slip boundary conditions on MHD Casson fluid across a stretched sheet in non-Darcy porous medium is the primary goal of the current work. By utilising similarity transformations, the controlling PDEs are changed into non-linear ODEs. Using an efficient shooting strategy, the transformed ODEs are solved. The numerical findings are in strong accord with the body of research. Graphs and tables are used to illustrate the impact of relevant factors.

**2. Mathematical Formulation**

Consider a steady two dimensional electrically conducting Casson fluid flow past a stretching sheet along positive *x*-direction. A transverse magnetic field of strength is applied along *y*-axis. The fluid flow is restricted towhich is caused by the linear stretching of the sheet with velocity  from a slit keeping the origin fixed as shown in Fig. 1. It is assumed that the magnetic Reynolds number of the fluid is very small so that the effect of induced magnetic field is neglected.



**Fig. 1** Flow geometry and coordinate system

The rheological equation of state for an isotropic and incompressible flow of Casson fluid is given by (Senapati et al. [16])



where is plastic dynamic viscosity of the non-Newtonian fluid, is the yield stress of the fluid, is the product of deformation rate with itself, is the component of the deformation rate and is a critical value of this product, based on the non-Newtonian model.

Under the above assumptions, the MHD boundary layer equations for steady flow of Casson fluid are given by

 (1)

 (2)

 (3)

The boundary conditions are

 (4)

where are velocity components in *x* and *y* directions respectively, is the magnetic field strength, is the kinematic viscosity, is the electrical conductivity, is the density, is the thermal conductivity, is the temperature of the fluid,  is the wall temperature, is the ambient temperature, is the specific heat, is the drag coefficient, is the permeability of the medium, and  is velocity slip factor .

Consider the stream function  such that and dimensionless variable is 

Therefore, the equation (1) is identically satisfied and the equations (2) - (4) become

 (5)

 (6)

 (7)

where is the magnetic parameter, is the porosity parameter, is the local inertia parameter, is the slip parameter, and is the Prandtl number.

The surface condition of practical interest is the local skin friction coefficient and local Nusselt number which are given by and respectively.

Here wall shear stress and wall heat flux  is the local Reynolds number.

**3. Results and Discussion**

 The dimensionless coupled nonlinear ODEs (4) and (5) are solved numerically by Runge-Kutta fourth order method with shooting technique using MATLAB software with step length and the error tolerance. In this method, the equations are reduced to a system of first order differential equations:











with the initial conditions

 

Now, the initial value problem is solved by suitably predicting the missing initial values by shooting technique. During calculation we fix the parameters as unless otherwise the values are mentioned. A comparison is made with previously published works of Hayat et al. [4] and Ibrahim and Shankar [5] as shown in Table 1. It is found that our numerical results are in good agreement.

**Table 1** Comparison of  for various values of when 

|  |  |  |  |
| --- | --- | --- | --- |
|  | Hayat et al. [4] | Ibrahim and Shankar [5] | Present results |
| 0.0 | 1.000000 | 1.0000 | 1.00000 |
| 0.1 | 0.872082 | 0.8721 | 0.87344 |
| 0.2 | 0.776377 | 0.7764 | 0.77770 |
| 0.5 | 0.591195 | 0.5912 | 0.71827 |
| 2 | 0.283981 | 0.2840 | 0.28493 |
| 5 | 0.144841 | 0.1448 | 0.14553 |
| 10 | 0.081249 | 0.0812 | 0.08596 |
| 20 | 0.043782 | 0.0438 | 0.04428 |
| 50 | 0.018634 | 0.0186 | 0.01875 |

Figs. 2 and 3 display the effects of magnetic parameter () and porosity parameteron velocity and temperature distribution respectively. It is observed that an increase  and  give rise to low flow rates due to additional resistive forces and consequently, boundary layer thickness decreases. But the opposite effects are observed in case of temperature distribution. The reason for this is that an increase in magnetic parameter causes an increase in electromagnetic force that restrains the fluid motion which in turn brings about the temperature rise leading to thicker thermal boundary layer.

Figs. 4 and 5 represent influences of inertia parameter  and Casson parameteron velocity and temperature distribution respectively. It is seen that the velocity profile declines with an increase in both the parameters whereas reverse trend is observed in case of temperature profile. Since, higher values of lead to decrease the yield stress.

Figs. 6 and 7 depict the impact of slip parameter on velocity and temperature distribution respectively. The velocity profile deceases with an increase in slip parameter but the fluid temperature increases. This result is well established with the work of Ibrahim and Shankar [5]. Fig. 8 shows the effect of Prandtl number on temperature profile. The higher values of, having lower thermal diffusivity contributes a reduction in fluid temperature and consequently, thermal boundary layer shrinks.



Fig. 2 Influences of and on velocity profile



Fig. 3 Influences of and on temperature profile



Fig. 4 Influences of and on velocity profile



Fig. 5 Influences of and on temperature profile



Fig. 6 Influence of  on velocity profile



Fig. 7 Influence of  on temperature profile



Fig. 8 Influence of  on temperature profile

**Table 2** Values of skin friction coefficientand local Nusselt number 

when 

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| 0.1 | 0.1 | 0.1 | 0.1 | 1 | -2.817804 | 0.598782 |
| 0.5 | 0.1 | 0.1 | 0.1 | 1 | -3.056013 | 0.578155 |
| 1 | 0.1 | 0.1 | 0.1 | 1 | -3.309591 | 0.555186 |
| 1 | 0.5 | 0.1 | 0.1 | 1 | -3.389258 | 0.548616 |
| 1 | 1 | 0.1 | 0.1 | 1 | -3.480784 | 0.5409936 |
| 1 | 1 | 0.3 | 0.1 | 1 | -1.984094 | 0.412997 |
| 1 | 1 | 0.5 | 0.1 | 1 | -1.403070 | 0.346950 |
| 1 | 1 | 0.5 | 0.5 | 1 | -1.127293 | 0.355697 |
| 1 | 1 | 0.5 | 2 | 1 | -0.969014 | 0.338624 |
| 1 | 1 | 0.5 | 2 | 2 | -0.969014 | 0.551084 |
| 1 | 1 | 0.5 | 2 | 3 | -0.969014 | 0.728621 |

Table 2 is computed to know the effects of different physical parameters on the skin friction coefficientand local Nusselt number. It is observed that skin friction coefficient increases as and increase whereas it decreases with an increase in and. Further, the rate of heat transfer declines with an increase in and, while it boosts as and increase.

**4. Conclusions**

From the present analysis the following conclusions are drawn:

* The thickness of momentum boundary layer decreases with an increase in magnetic field parameters  and.
* The slip parameter increases the skin friction coefficient but decreases the rate of heat transfer at the surface of the sheet.
* Thermal boundary layer thickness decreases with an increase in values of Prandtl number while reverse tend is observed in case of slip parameter.

**References**

[1] B. C. Sakiadis, Boundary layer behavior on continuous solid surface: II. The boundary layer on a continuous flat surface, J Am Ins Chem Eng, 7 (1961) 221-5.

[2] T. Fang, J. Zhang, S. Yao, Slip MHD viscous flow over a stretching sheet-an exact solution. Commun Non-linear Sci Numer Simul, 14 (2009) 3731-7.

[3] M. Mahantesh, K. Vajravelu, M. S. Abel, M. N. Siddalingappa, Second order slip flow and heat transfer over a stretching sheet with non-linear Navier boundary condition, Int J Therm Sci, 58 (2012) 142-50.

[4] T. Hayat, M. Qasim, S. Mesloub, MHD flow and heat transfer over permeable stretching sheet with slip conditions, Int J Numer Meth Fluid, 66 (2011) 963-75.

[5] W. Ibrahim, B. Shankar, MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions, Computers & Fluids, 75 (2013) 1-10.

[6] O. D. Makinde, F. Mabood, W. A. Khan, and M. S. Tshehla, MHD flow of a variable viscosity nanofluid over a radially stretching convective surface with radiative heat, Journal of Molecular Liquids, vol. 219, pp. 624–630, 2016.

[7] K. Swain, S. K. Parida, G. C. Dash, MHD Heat and Mass Transfer on Stretching Sheet with Variable Fluid Properties in Porous Medium, AMSE, Modelling B, 86 (2017) 706-726.

[8] K. Swain, S. K. Parida, G. C. Dash, Effects of non-uniform heat source/sink and viscous dissipation on MHD boundary layer flow of Williamson nanofluid through porous medium, Defect and Diffusion Forum, 389 (2018) 110-127.

[9] K. Swain, S. K. Parida, G. C. Dash, Thermal slip effect on MHD convective nanofluid flow over a vertical plate embedded in a porous medium, European Journal of Electrical Engineering. 20 (2018) 215-233.

[10] K. Swain, S. K. Parida, G. C. Dash, Higher Order Chemical Reaction on MHD Nanofluid Flow with Slip Boundary Conditions: A Numerical Approach, Mathematical Modelling of Engineering Probelems, 6 (2019) 293-299.

[11] M. Y. Abou-Zeid, A. A. Shaaban, and M. Y. Alnour, Numerical treatment and global error estimation of natural convective effects on gliding motion of bacteria on a power-lawnanoslime through a non-Darcy porous medium, Journal of Porous Media, 18 (2015) 1091-1106.

[12] N. Casson, In Reheology of Dipersed System, Peragamon Press, Oxford, UK, 1959.

[13] S.M. Ibrahim, P.V. Kumar, G. Lorenzini, E. Lorenzini, F. Mabood, Numerical study of the onset of chemical reaction and heat source on dissipative MHD stagnation point flow of Casson nanofluid over a nonlinear stretching sheet with velocity slip and convective boundary conditions, Journal of Engineering Thermophysics. 26 (2017) 256-271.

[14] K. Bhattacharyya, T. Hayat, and A. Alsaedi, Analytic solution for magnetohydrodynamic boundary layer flow of Casson fluid over a stretching/shrinking sheet with wall mass transfer, Chinese PhysicsB, 22 (2013) Article ID 024702.

[15] S. Mukhopadhyay, Casson fluid flow and heat transfer over a nonlinearly stretching, Chinese Physics B, 22 (2013) Article ID 074701.

[16] M. Senapati, K. Swain, S.K. Parida, Numerical analysis of three-dimensional MHD flow of Casson nanofluid past an exponentially stretching sheet, Karbala International Journal of Modern Science, 6 (2020) Article 13.