$H\_{k} $**Cordial Labeling of Path, Star and Cycle Graphs**

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# **Abstract**

In this paper we investigate $H\_{k} $cordial labeling of star, path, cycle and use operation such as subdivision, super subdivision and $H $- super subdivision on it, i.e. $S\left(P\_{n}\right), SS\left(P\_{n}\right) , HSS\left(P\_{n}\right), $ $S\left(K\_{1,n}\right), SS\left(K\_{1,n}\right), HSS\left(K\_{1,n}\right), $ $S\left(C\_{n}\right), SS\left(C\_{n}\right), HSS\left(C\_{n}\right)$.

***Keywords*-** $H $cordial labeling, $H\_{k} $cordial labeling, Subdivision, Super subdivision, $H $-Super subdivision of graphs.

# **Introduction**

In the present work we contemplate a finite graph which is connected and undirected. We refer to a dynamic survey of graph labeling by Gallian (2020) for detailed survey on graph labeling. For all other standard terminology and notations we refer to Gross and Yellen[4]. A labeling of a graph $ G=(V,E)$ is a mapping that carries vertices, edges or both to the set of labels (usually to the positive or non-negative integers).

A graph $G=(V,E)$ is said to be $H$ cordial graph if there exists a mapping $f $from edge set to $\left\{-1, 1\right\}$ such that induced mapping $f^{\*}$ from vertex set to $\left\{-k, k\right\} $defined by $f^{\*}\left(v\right) = \sum\_{e\in I(v)}^{}f(e)$ , where $I(v)$ is the set of all edges incident to vertex$ v$, satisfies the cordiality conditions $\left|e\_{f}\left(1\right)-e\_{f}\left(-1\right)\right|\leq 1$ and $\left|v\_{f^{\*}}\left(k\right)-v\_{f^{\*}}\left(-k\right)\right|\leq 1$. Map $ f $ is called $H$ cordial labeling of $G$. By extending the concept a graph$ $is $H\_{k}$ cordial graph if there exists a mapping $f $from edge set to $\{\pm 1,\pm 2,…,\pm k\}$ such that the induced mapping $f^{\*}$ from vertex set to $\left\{\pm 1,\pm 2,…,\pm k\right\} $ defined by $f^{\*}\left(v\right) = \sum\_{e\in I(v)}^{}f(e)$ , where $I(v)$ is the set of all edges incident to vertex$ v$, satisfies the cordiality conditions $\left|e\_{f}(i)-e\_{f}(-i)\right|\leq 1 $ and$ \left|v\_{f^{\*}}(i)-v\_{f^{\*}}(-i)\right|\leq 1 $for $1\leq i\leq k . $ Map $f $is called $H\_{k} $cordial labeling of $G $ and graph $G $ is called $H\_{k} $cordial graph [5].

Barycentric subdivision of graph $G $is denoted as$ S(G)$, obtained by subdividing every edge of graph $ G$. [10]. The super subdivision of any graph $ G$ denoted by $SS\left(G\right)$ is obtained from graph by replacing every edge of graph by complete bipartite graph $K\_{2,m}$ (where $m$ is positive integer)[8].

# **II. Main Result**

***Theorem 2.1*** The star graph $ K\_{1,n} $ $(n\geq 2 )$is $ H\_{2} $cordial if $n$ is even.$ $

***Proof*:** Let $ V\left(K\_{1,n}\right)= \left\{u\_{0},u\_{i} :1 \leq i\leq n\right\}$ and $E( K\_{1,n})=\left\{u\_{0}u\_{i}:1 \leq i\leq n\right\}$ , where $u\_{0}$ is apex vertex.

Consider a function $f:E( K\_{1,n})\rightarrow \{-2,-1,1,2\}$ defined as

$f\left(u\_{0}u\_{1}\right)=-2$

$f\left(u\_{0}u\_{i}\right)= (-1)^{i}$ ;$ 2\leq i\leq n$.

**Table 1**

|  |  |  |
| --- | --- | --- |
| $$n\geq 2$$ | Edge Conditions | Vertex Conditions |
| $n$ is even | $$e\_{f}\left(1\right)=\frac{n}{2},e\_{f}\left(-1\right)=\frac{n-2}{2}$$$$e\_{f}\left(2\right)=0,e\_{f}\left(-2\right)=1$$ | $$v\_{f^{\*}}\left(1\right)=\frac{n}{2}=v\_{f^{\*}}\left(-1\right)$$$$v\_{f^{\*}}\left(2\right)=0,v\_{f^{\*}}\left(-2\right)=1$$ |

For$ i=1,2$. The $K\_{1,n}$ satisfies the condition $\left|e\_{f}\left(i\right)-e\_{f}(-i)\right|\leq 1$ and$ \left|v\_{f}\left(i\right)-v\_{f}(-i)\right|\leq 1$.

Hence, $K\_{1,n} $ is $H\_{2} $cordial.

***Illustration 2.2***$ $Figure shows that $K\_{1,6} $is $H\_{2} $cordial graph.



***Theorem 2.3*** Star graph $K\_{1,n} $ $\left(n\geq 2 \right) $is $ H\_{3} $cordial.

***Proof*:** Let $ V\left(K\_{1,n}\right)= \left\{u\_{0},u\_{i} :1 \leq i\leq n\right\}$ and $E( K\_{1,n})=\left\{u\_{0}u\_{i}:1 \leq i\leq n\right\}$, where $u\_{0}$ is apex vertex.

**Type 1:**  $n $is even, $K\_{1,n}$ is $H\_{2} $cordial from Theorem 2.1. Hence it is also admits $H\_{3} $cordial labeling.

**Type 2:**  $n $is odd.

Consider a function $f:E( K\_{1,n})\rightarrow \{-3,-2,-1, 1, 2, 3\}$ defined as

$f\left(u\_{0}u\_{1}\right)=-2$

$f\left(u\_{0}u\_{1}\right)=-2$ $ $

$f\left(u\_{0}u\_{2}\right)=3$

$f\left(u\_{0}u\_{i}\right)= (-1)^{i+1}$ ;$ 3\leq i\leq n$

**Table 2**

|  |  |  |
| --- | --- | --- |
| $$n\geq 2$$ | Edge Conditions | Vertex Conditions |
| $n$ is odd | $$e\_{f}\left(1\right)=\frac{n-1}{2},e\_{f}\left(-1\right)=\frac{n-3}{2}$$$$e\_{f}\left(2\right)=0,e\_{f}\left(-2\right)=1$$$$e\_{f}\left(3\right)=1,e\_{f}\left(-3\right)=0$$ | $$v\_{f^{\*}}\left(1\right)=\frac{n-1}{2},v\_{f^{\*}}\left(-1\right)=\frac{n-3}{2}$$$$v\_{f^{\*}}\left(2\right)=1=v\_{f^{\*}}\left(-2\right)$$$$v\_{f^{\*}}\left(3\right)=1,v\_{f^{\*}}\left(-3\right)=0$$ |

For $i=1,2,3$, the graph satisfies the condition $\left|e\_{f}\left(i\right)-e\_{f}(-i)\right|\leq 1$ and$ \left|v\_{f}\left(i\right)-v\_{f}(-i)\right|\leq 1$.

Hence, $K\_{1,n} $ is $H\_{3} $- cordial.

***Illustration 2.4***$K\_{1,5} $is $H\_{3} $cordial as shown in Figure.



***Theorem 2.5*** The barycentric subdivision graph of a star ( $S(K\_{1,n}) (n\geq 2 )$) is $ H\_{2} $cordial if $n$ is odd.

***Proof*:** Let $V( S(K\_{1,n}))=\left\{u\_{i},u^{'}\_{i},u\_{0};1\leq i\leq n\right\}$ and$ E( S(K\_{1,n}))=\left\{u\_{0}u'\_{i},u\_{i}u^{'}\_{i} ;1\leq i\leq n\right\}$.

Consider a function $f:E( S(K\_{1,n}))\rightarrow \{-2,-1,1,2\}$ defined as

$$f\left(u\_{0}u'\_{1}\right)=-2$$

$f\left(u\_{0}u'\_{i}\right)= \left(-1\right)^{i+1}$;$ 2\leq i\leq n$

$f\left(u\_{i}u'\_{i}\right)= (-1)^{i+1}$ ;$ 1\leq i\leq n$

**Table 3**

|  |  |  |
| --- | --- | --- |
| $$n\geq 2$$ | Edge Conditions | Vertex Conditions |
| $n$ is odd | $$e\_{f}\left(1\right)=n,e\_{f}\left(-1\right)=n+1$$$$e\_{f}\left(2\right)=0,e\_{f}\left(-2\right)=1$$ | $$v\_{f^{\*}}\left(1\right)=\frac{n+1}{2}=v\_{f^{\*}}\left(-1\right)$$$$v\_{f^{\*}}\left(2\right)=\frac{n-1}{2},v\_{f^{\*}}\left(-2\right)=\frac{n+1}{2}$$ |

Hence, $S(K\_{1,n}) $ is $H\_{2}$ cordial.

***Illustration 2.6***$S(K\_{1,5}) $is $H\_{2} $cordial as shown in Figure.



***Theorem 2.7*** The barycentric subdivision graph of a star $S(K\_{1,n}) (n\geq 2 )$ is $ H\_{3} $cordial.

***Proof*:** Let $V(S(K\_{1,n}))=\left\{u\_{i},u^{'}\_{i},u\_{0};1\leq i\leq n\right\}$ and $ E(S(K\_{1,n}))=\left\{u\_{0}u'\_{i},u\_{i}u^{'}\_{i} ;1\leq i\leq n\right\}$.

**Type 1:** $n$ is odd.

$S(K\_{1,n}) $is $H\_{2} $cordial from Theorem 2.5, it is also admits $H\_{3} $cordial.

**Type 2:** $n$ is even.

 Consider a function $ f:E(S(K\_{1,n}))\rightarrow \{-2,-1,1,2\}$ defined as

$$f\left(u\_{0}u'\_{1}\right)=2$$

$f\left(u\_{0}u'\_{i}\right)= \left(-1\right)^{i}$;$ 2\leq i\leq n$

$f\left(u\_{i}u'\_{i}\right)= (-1)^{i}$ ;$ 1\leq i\leq n$

**Table 4**

|  |  |  |
| --- | --- | --- |
| $$n\geq 2$$ | Edge Conditions | Vertex Conditions |
| $n$ is even | $$e\_{f}\left(1\right)=n,e\_{f}\left(-1\right)=n-1$$$$e\_{f}\left(2\right)=1,e\_{f}\left(-2\right)=0$$ | $$v\_{f^{\*}}\left(1\right)=\frac{n+2}{2},v\_{f^{\*}}\left(-1\right)=\frac{n}{2}$$$$v\_{f^{\*}}\left(2\right)=\frac{n}{2},v\_{f^{\*}}\left(-2\right)=\frac{n-2}{2}$$$$v\_{f^{\*}}\left(3\right)=1,v\_{f^{\*}}\left(-3\right)=0$$ |

Hence, $S(K\_{1,n}) $ is $H\_{3} $cordial.

***Illustration 2.8*** $H\_{3} $cordial labeling of $S(K\_{1,6}) $is demonstrated in Figure.



***Theorem 2.9***Super subdivision of star graph $ SS(K\_{1,n})(n\geq 2)$ is $H\_{3}$ cordial.

***Proof*:** Let $V( SS(K\_{1,n}))=\left\{u\_{i},u\_{ij},u\_{0};1\leq i\leq n ,1\leq j\leq m\right\}$and$ E( SS(K\_{1,n}))=\left\{u\_{0}u\_{ij},u\_{ij}u\_{i};1\leq i\leq n,1\leq j\leq m\right\}$, where $u\_{0}$ is apex vertex.

Consider a function $f:E( SS(K\_{1,n}))\rightarrow \{-2,-1,1,2\}$ defined as

**Type 1:** $ m $is even and $n $is odd

$$f\left(u\_{0}u\_{11}\right)=-2$$

$$ f\left(u\_{0}u\_{12}\right)=1$$

$$f\left(u\_{11}u\_{1}\right)=-1 $$

$$f\left(u\_{12}u\_{1}\right)=2$$

$f\left(u\_{0}u\_{ij}\right)=f\left(u\_{ij}u\_{i}\right)=(-1)^{i}$ ; $2\leq i\leq n,j=1,2$

$f\left(u\_{0}u\_{ij}\right)=f\left(u\_{ij}u\_{i}\right)=(-1)^{j}$ ; $3\leq j\leq m, 1\leq i\leq n$.

**Table 5**

|  |  |  |
| --- | --- | --- |
| $$n\geq 2$$ | Edge Conditions | Vertex Conditions |
| $m$ even$n$ odd | $$e\_{f}\left(1\right)=mn-1=e\_{f}\left(-1\right)$$$$e\_{f}\left(2\right)=1=e\_{f}\left(-2\right)$$ | $$v\_{f^{\*}}\left(1\right)=1=v\_{f^{\*}}\left(-1\right)$$$$v\_{f^{\*}}\left(2\right)=\frac{\left(m+1\right)n-3}{2}=v\_{f^{\*}}\left(-2\right)$$$$v\_{f^{\*}}\left(3\right)=1=v\_{f^{\*}}\left(-3\right)$$ |

**Type 2:** $ m $and $n $both are even

$$f\left(u\_{0}u\_{11}\right)=-2$$

$$f\left(u\_{0}u\_{12}\right)=f\left(u\_{11}u\_{1}\right)=f\left(u\_{12}u\_{1}\right)=-1$$

$f\left(u\_{0}u\_{ij}\right)=f\left(u\_{ij}u\_{i}\right)=(-1)^{i}$ ; $2\leq i\leq n, j=1,2$

$f\left(u\_{0}u\_{ij}\right)=f\left(u\_{ij}u\_{i}\right)=(-1)^{j}$ ; $3\leq j\leq m, 1\leq i\leq n$

**Table 6**

|  |  |  |
| --- | --- | --- |
| $$n\geq 2$$ | Edge Conditions | Vertex Conditions |
| $m$ even$n$ even | $$e\_{f}\left(1\right)=mn, e\_{f}\left(-1\right)=mn-1$$$$e\_{f}\left(2\right)=1,e\_{f}\left(-2\right)=0$$ | $$v\_{f^{\*}}\left(1\right)=1,v\_{f^{\*}}\left(-1\right)=0$$$$v\_{f^{\*}}\left(2\right)=\frac{n\left(m+1\right)}{2}$$$$v\_{f^{\*}}\left(-2\right)=\frac{n\left(m+1\right)-2}{2}$$$$v\_{f^{\*}}\left(3\right)=1,v\_{f^{\*}}\left(-3\right)=0$$ |

**Type 3:** $m $and $n $ both are odd

$$f\left(u\_{0}u\_{11}\right)=-2$$

$$f\left(u\_{11}u\_{1}\right)=1$$

$f\left(u\_{0}u\_{i1}\right)=f\left(u\_{i1}u\_{i}\right)=(-1)^{i+1}$ ; $2\leq i\leq n$

$f\left(u\_{0}u\_{ij}\right)=f\left(u\_{ij}u\_{i}\right)=(-1)^{j}$ ; $2\leq j\leq m, 1\leq i\leq n$

**Table 7**

|  |  |  |
| --- | --- | --- |
| $$n\geq 2$$ | Edge Conditions | Vertex Conditions |
| $m$ odd$n$ odd | $$e\_{f}\left(1\right)=mn,e\_{f}\left(-1\right)=mn-1$$$$e\_{f}\left(2\right)=0,e\_{f}\left(-2\right)=1$$ | $$v\_{f^{\*}}\left(1\right)=\frac{n+1}{2}=v\_{f^{\*}}\left(-1\right)$$$$v\_{f^{\*}}\left(2\right)=\frac{mn-1}{2},v\_{f^{\*}}\left(-2\right)=\frac{mn-1}{2}$$ |

**Type 4:** $ m $is odd and $n $is even

$$f\left(u\_{0}u\_{11}\right)=2$$

$$f\left(u\_{11}u\_{1}\right)=-1$$

$f\left(u\_{0}u\_{i1}\right)=f\left(u\_{i1}u\_{i}\right)=(-1)^{i}$ ; $2\leq i\leq n$

$f\left(u\_{0}u\_{ij}\right)=f\left(u\_{ij}u\_{i}\right)=(-1)^{j}$ ; $2\leq j\leq m, 1\leq i\leq n$

**Table 8**

|  |  |  |
| --- | --- | --- |
| $$n\geq 2$$ | Edge Conditions | Vertex Conditions |
| $m$ odd$n$ even | $$e\_{f}\left(1\right)=mn,e\_{f}\left(-1\right)=mn-1$$$$e\_{f}\left(2\right)=1,e\_{f}\left(-2\right)=0$$ | $$v\_{f^{\*}}\left(1\right)=\frac{n+2}{2},v\_{f^{\*}}\left(-1\right)=\frac{n}{2}$$$$v\_{f^{\*}}\left(2\right)=\frac{mn}{2},v\_{f^{\*}}\left(-2\right)=\frac{mn-2}{2}$$$$v\_{f^{\*}}\left(3\right)=1,v\_{f^{\*}}\left(-3\right)=0$$ |

Hence, $ SS(K\_{1,n}) $ is $H\_{3}$ cordial.

***Illustration 2.10***$ SS(K\_{1,4}) $with$m=3 $is $H\_{3}$ cordial shown in Figure.



***Theorem 2.11***The $H $-super subdivision of path $ HSS(K\_{1,n})(n\geq 2)$ $H\_{3}$cordial.

***Proof*:** Let $V\left( HSS\left(K\_{1,n}\right)\right)=\left\{u\_{i},u\_{ij},u\_{0};1\leq i\leq n ,1\leq j\leq 4\right\}$ and $ E\left(HSS\left(K\_{1,n}\right)\right) =\left\{u\_{0}u\_{i1},u\_{i}u\_{i3},u\_{i1}u\_{i3},u\_{i1}u\_{i2},u\_{i3}u\_{i4};1\leq i\leq n\right\}$, where $u\_{0}$ is apex vertex.

**Type 1:** $n $is odd, consider a function $f:E\left(HSS\left(K\_{1,n}\right)\right)\rightarrow \{-1,1\}$ defined as

$$f\left(u\_{0}u\_{11}\right)=f\left(u\_{11}u\_{12}\right)=1$$

$$f\left(u\_{1}u\_{13}\right)=f\left(u\_{13}u\_{14}\right)=-1$$

$$f\left(u\_{i1}u\_{i3}\right)=(-1)^{i};1\leq i\leq n$$

$f\left(u\_{0}u\_{i1}\right)=f\left(u\_{i1}u\_{i2}\right)=f\left(u\_{i}u\_{i3}\right)=f\left(u\_{i3}u\_{i4}\right)=(-1)^{i+1};2\leq i\leq n$.

**Table 9**

|  |  |  |
| --- | --- | --- |
| $$n\geq 2$$ | Edge Conditions | Vertex Conditions |
| $n $is odd | $$e\_{f}\left(1\right)=\frac{5n-1}{2},e\_{f}\left(-1\right)=\frac{5n+1}{2}$$ | $$v\_{f^{\*}}\left(1\right)=\frac{5n+1}{2},v\_{f^{\*}}\left(-1\right)=\frac{5n-1}{2}$$$$v\_{f^{\*}}\left(3\right)=0, v\_{f^{\*}}\left(-3\right)=1$$ |

Hence $f$ satisfies the conditions of $H\_{3}$ cordial labeling in this Type and hence the graph under consideration is $H\_{3}$ cordial graph, when $n$ is odd.

**Type 2:** $n $is even, consider a function $f:E\rightarrow \{-2,-1,1,2\}$ defined as

$$f\left(u\_{0}u\_{11}\right)=2$$

$$f\left(u\_{0}u\_{21}\right)=-1$$

$$f\left(u\_{i}u\_{i3}\right)=(-1)^{i+1};i=1,2$$

$$f\left(u\_{i1}u\_{i2}\right)=f\left(u\_{i3}u\_{i4}\right)=(-1)^{i};i=1,2$$

$$f\left(u\_{i1}u\_{i3}\right)=(-1)^{i+1};1\leq i\leq n$$

$$f\left(u\_{0}u\_{i1}\right)=f\left(u\_{i1}u\_{i2}\right)=f\left(u\_{i}u\_{i3}\right)=f\left(u\_{i3}u\_{i4}\right)=(-1)^{i};3\leq i\leq n.$$

**Table 10**

|  |  |  |
| --- | --- | --- |
| $$n\geq 2$$ | Edge Conditions | Vertex Conditions |
| $n $is even | $$e\_{f}\left(1\right)=\frac{5n-2}{2},e\_{f}\left(-1\right)=\frac{5n}{2}$$$$e\_{f}\left(2\right)=1,e\_{f}\left(-2\right)=0$$ | $$v\_{f^{\*}}\left(1\right)=\frac{5n}{2}=v\_{f^{\*}}\left(-1\right)$$$$v\_{f^{\*}}\left(2\right)=1,v\_{f^{\*}}\left(-2\right)=0$$ |

In this Type $f$ satisfies the conditions of $H\_{2}$ cordial labeling and hence the graph under consideration is $H\_{2}$ cordial graph, when $n$ is even.

Hence, $ HSS(K\_{1,n}) $ is $H\_{3} $cordial as per above Types.

***Illustration 2.12***$ HSS(K\_{1,4}) $is $H\_{2} $cordial shown in Figure.



***Theorem 2.13***Path $ P\_{n}(n\geq 3)$ is $H\_{3}$cordial.

***Proof*:** Let $P\_{n}$ be the path $ u\_{1}, u\_{2}, …,u\_{n}$.

Consider a function $f:E(P\_{n})\rightarrow \{-2,-1,1,2\}$ defined as

$$f\left(u\_{i}u\_{i+1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \left⌈\frac{n}{2}\right⌉-1\\2&;i=\left⌈\frac{n}{2}\right⌉ \\-1&;Otherwise \end{matrix}\right.$$

**Table 11**

|  |  |  |
| --- | --- | --- |
| $$n\geq 3$$ | Edge Conditions | Vertex Conditions |
| $n$ is even | $$e\_{f}\left(1\right)=\frac{n-2}{2}=e\_{f}\left(-1\right)$$$$e\_{f}\left(2\right)=1,e\_{f}\left(-2\right)=0$$ | $$v\_{f^{\*}}\left(1\right)=2,v\_{f^{\*}}\left(-1\right)=1$$$$v\_{f^{\*}}\left(2\right)=\frac{n-4}{2}=v\_{f^{\*}}\left(-2\right)$$$$v\_{f^{\*}}\left(3\right)=1,v\_{f^{\*}}\left(-3\right)=0$$ |
| $n$ is odd | $$e\_{f}\left(1\right)=\frac{n-1}{2},e\_{f}\left(-1\right)=\frac{n-3}{2}$$$$e\_{f}\left(2\right)=1,e\_{f}\left(-2\right)=0$$ | $$v\_{f^{\*}}\left(1\right)=2,v\_{f^{\*}}\left(-1\right)=1$$$$v\_{f^{\*}}\left(2\right)=\frac{n-4}{2},v\_{f^{\*}}\left(-2\right)=\frac{n-6}{2}$$$$v\_{f^{\*}}\left(3\right)=1,v\_{f^{\*}}\left(-3\right)=0$$ |

Hence, $ P\_{n} $ is $H\_{3} $cordial.

***Illustration 2.14*** $H\_{3} $cordial labeling of $ P\_{6} $ is as shown in below Figure.



***Remarks 2.15***

Consider path$ P\_{n}$. As per barycentric subdivision of $ P\_{n}(n\geq 2)$ is again a path $ P\_{2n-1}$ which is also is $H\_{3}$cordial as per Theorem 2.13. Hence we have the following.

***Theorem 2.16***The barycentricsubdivision of path $ S(P\_{n})(n\geq 2)$ is $H\_{3} $cordial.

***Theorem 2.17***The super subdivision of path $ SS(P\_{n})(n\geq 3)$ is $H\_{3}$cordial.

***Proof*:** Let $V( SS(P\_{n}))=\left\{u\_{i},u\_{ij},u\_{n};1\leq i\leq n-1 , 1\leq j\leq m\right\}$and$ E( SS(P\_{n}))=\left\{u\_{i}u\_{ij},u\_{ij}u\_{i+1}; 1\leq i\leq n-1, 1\leq j\leq m\right\}$.

Consider a function $f:E\rightarrow \{-2,-1,1,2\}$ defined as

**Type 1:** $ m $is even and $ n $is odd.

$$f\left(u\_{i}u\_{i1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n-3}{2}\\2& ;i= \frac{n-1}{2} \\-1&;Otherwise \end{matrix}\right.$$

$$f\left(u\_{i1}u\_{i+1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n-3}{2}\\-1&;Otherwise \end{matrix}\right.$$

$f\left(u\_{i}u\_{i2}\right)=f\left(u\_{i2}u\_{i+1}\right)=(-1)^{i+1}$ ; $1\leq i\leq n-1$

$f\left(u\_{i}u\_{ij}\right)=f\left(u\_{ij}u\_{i+1}\right)=(-1)^{j}$ ; $3\leq j\leq m.$

**Table 12**

|  |  |  |
| --- | --- | --- |
| $$n\geq 3$$ | Edge Conditions | Vertex Conditions |
| $m $is even$n$ is odd | $$e\_{f}\left(1\right)=m\left(n-1\right),$$$$e\_{f}\left(-1\right)=m\left(n-1\right)-1$$$$e\_{f}\left(2\right)=1,e\_{f}\left(-2\right)=0$$ | $$v\_{f^{\*}}\left(1\right)=1,v\_{f^{\*}}\left(-1\right)=0$$$$v\_{f^{\*}}\left(2\right)=\frac{\left(m+1\right)\left(n-1\right)}{2}$$$$v\_{f^{\*}}\left(-2\right)=\frac{\left(m+1\right)\left(n-1\right)-2}{2}$$$$v\_{f^{\*}}\left(3\right)=1,v\_{f^{\*}}\left(-3\right)=0$$ |

**Type 2:** $m $ and $ n $both are even

$$f\left(u\_{i}u\_{i1}\right)=f\left(u\_{i1}u\_{i+1}\right)=\left\{\begin{matrix}1&;2\leq i\leq \frac{n}{2} \\-1&;Otherwise\end{matrix}\right.$$

$$f\left(u\_{i}u\_{ij}\right)=f\left(u\_{ij}u\_{i+1}\right)=\left\{\begin{matrix}1&;j=1,2 ;i=1 \\-1&;j=1,2 ;i=n-1\end{matrix}\right.$$

$$f\left(u\_{i}u\_{i2}\right)=\left\{\begin{matrix}-2&;2\leq i\leq \frac{n}{2} \\2&;Otherwise\end{matrix}\right.$$

$$f\left(u\_{i2}u\_{i+1}\right)=\left\{\begin{matrix}-1&;2\leq i\leq \frac{n}{2} \\1&;Otherwise\end{matrix}\right.$$

$f\left(u\_{i}u\_{ij}\right)=f\left(u\_{ij}u\_{i+1}\right)=(-1)^{j}$ ; $3\leq j\leq m.$

**Table 13**

|  |  |  |
| --- | --- | --- |
| $$n\geq 3$$ | Edge Conditions | Vertex Conditions |
| $m $is even$n$ is even | $$e\_{f}\left(1\right)=\frac{\left(2m-1\right)\left(n-1\right)+3}{2}$$$$e\_{f}\left(-1\right)=\frac{\left(2m-1\right)\left(n-1\right)+1}{2}$$$$e\_{f}\left(2\right)=\frac{n-4}{2},e\_{f}\left(-2\right)=\frac{n-2}{2}$$ | $$v\_{f^{\*}}\left(1\right)=\frac{n-2}{2},v\_{f^{\*}}\left(-1\right)=\frac{n-4}{2}$$$$v\_{f^{\*}}\left(2\right)=\frac{\left(m-1\right)\left(n-1\right)+5}{2}=v\_{f^{\*}}\left(-2\right)$$$$v\_{f^{\*}}\left(3\right)=\frac{n-4}{2},v\_{f^{\*}}\left(-3\right)=\frac{n-2}{2}$$ |

**Type 3:** $m $is odd and $n\geq 3$

$$f\left(u\_{i}u\_{i1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \left⌈\frac{n}{2}\right⌉-1\\2&;i=\left⌈\frac{n}{2}\right⌉ \\-1&;Otherwise \end{matrix}\right.$$

$$f\left(u\_{i1}u\_{i+1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \left⌈\frac{n}{2}\right⌉-1\\-1&;Otherwise \end{matrix}\right.$$

$f\left(u\_{i}u\_{ij}\right)=f\left(u\_{ij}u\_{i+1}\right)=(-1)^{j}$ ; $2\leq j\leq m.$

**Table 14**

|  |  |  |
| --- | --- | --- |
| $n\geq 3,m$ odd | Edge Conditions | Vertex Conditions |
| $n$ is even | $$e\_{f}\left(1\right)=m\left(n-1\right)-1,$$$$e\_{f}\left(-1\right)=m\left(n-1\right)$$$$e\_{f}\left(2\right)=1,e\_{f}\left(-2\right)=0$$ | $$v\_{f^{\*}}\left(1\right)=2,v\_{f^{\*}}\left(-1\right)=1$$$$v\_{f^{\*}}\left(2\right)=\frac{\left(m+1\right)\left(n-1\right)-4}{2}$$$$v\_{f^{\*}}\left(-2\right)=\frac{\left(m+1\right)\left(n-1\right)-2}{2}$$$$v\_{f^{\*}}\left(3\right)=1,v\_{f^{\*}}\left(-3\right)=0$$ |
| $n$ is odd | $$e\_{f}\left(1\right)=m\left(n-1\right),$$$$e\_{f}\left(-1\right)=m\left(n-1\right)-1$$$$e\_{f}\left(2\right)=1,e\_{f}\left(-2\right)=0$$ | $$v\_{f^{\*}}\left(1\right)=2,v\_{f^{\*}}\left(-1\right)=1$$$$v\_{f^{\*}}\left(2\right)=\frac{\left(m+1\right)\left(n-1\right)-2}{2}$$$$v\_{f^{\*}}\left(-2\right)=\frac{\left(m+1\right)\left(n-1\right)-4}{2}$$$$v\_{f^{\*}}\left(3\right)=1,v\_{f^{\*}}\left(-3\right)=0$$ |

Hence, $ SS(P\_{n}) $ is $H\_{3}$cordial.

***Illustration 2.18***$ SS(P\_{4}) $with$m=5 $is $H\_{3}$ cordial shown in Figure.



***Theorem 2.19***The $H $-super subdivision of path $ HSS(P\_{n})(n\geq 2)$ is $ H\_{3}$cordial.

***Proof*:** Let $V(HSS(P\_{n}))=\left\{u\_{i},u\_{ij};1\leq i\leq n ,1\leq j\leq 4\right\}$ and $E\left(HSS\left(P\_{n}\right)\right)=\left\{u\_{i}u\_{i1},u\_{i3}u\_{i+1},u\_{i1}u\_{i3},u\_{i1}u\_{i2},u\_{i3}u\_{i4};1\leq i\leq n-1\right\}.$

Consider a function $f:E(HSS(P\_{n}))\rightarrow \{-2,-1,1,2\}$ defined as

**Type 1:**  $ n $is odd.

$$f\left(u\_{i}u\_{i1}\right)=f\left(u\_{i1}u\_{i3}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n-1}{2}\\-1&;Otherwise \end{matrix}\right.$$

$$f\left(u\_{i+1}u\_{i3}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n-3}{2}\\2&;i=\frac{n-1}{2}\\-1&;Otherwise\end{matrix}\right.$$

$$f\left(u\_{i1}u\_{i2}\right)=f\left(u\_{i3}u\_{i4}\right)=\left\{\begin{matrix}-1&;1\leq i\leq \frac{n-1}{2}\\1&;Otherwise \end{matrix}\right.$$

**Table 15**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| $n $is odd | $$e\_{f}\left(1\right)=\frac{5n-7}{2},e\_{f}\left(-1\right)=\frac{5n-5}{2}$$$$e\_{f}\left(2\right)=1,e\_{f}\left(-2\right)=0$$ | $$v\_{f^{\*}}\left(1\right)=2n-1=v\_{f^{\*}}(-1)$$$$v\_{f^{\*}}\left(2\right)=\frac{n-1}{2}, v\_{f^{\*}}\left(-2\right)=\frac{n-3}{2}$$ |

Hence $f$ satisfies the conditions $H\_{2} $cordial labeling in this Type.

**Type 2:** $ n $is even.

$$f\left(u\_{i}u\_{i1}\right)=f\left(u\_{i+1}u\_{i3}\right)= (-1)^{i};2\leq i\leq n-1$$

$$f\left(u\_{1}u\_{11}\right)=1$$

$$f\left(u\_{n}u\_{\left(n-1\right)3}\right)=-1$$

$$f\left(u\_{i1}u\_{i3}\right)=f\left(u\_{i3}u\_{i4}\right)=(-1)^{i};1\leq i\leq n-1$$

$$f\left(u\_{i1}u\_{i2}\right)=\left(-1\right)^{i+1};1\leq i\leq n-1.$$

**Table 16**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| $n $is even | $$e\_{f}\left(1\right)=\frac{5n-6}{2},e\_{f}\left(-1\right)=\frac{5n-4}{2}$$ | $$v\_{f^{\*}}\left(1\right)=2n-1,v\_{f^{\*}}\left(-1\right)=2n-2$$$$v\_{f^{\*}}\left(2\right)=\frac{n-2}{2}=v\_{f^{\*}}\left(-2\right)$$$$v\_{f^{\*}}\left(3\right)=0, v\_{f^{\*}}\left(-3\right)=1$$ |

Hence $f$ satisfies the conditions $H\_{3}$cordial labeling in this Type.

 Hence, $ HSS(P\_{n}) $ is $H\_{3} $cordial graph.

***Illustration 2.20***$ $ $H\_{3} $cordial labeling of $ HSS(P\_{5}) $as shown in below Figure.



***Theorem 2.21***Cycle$ C\_{n}(n\geq 4)$ is $H\_{3}$ cordial.

***Proof*:** Let $V(C\_{n})=\left\{u\_{i};1\leq i\leq n \right\}$ and $ E(C\_{n})=\left\{u\_{i}u\_{i+1},u\_{1}u\_{n};1\leq i\leq n-1\right\}$.

Consider a function $f:E(C\_{n})\rightarrow \{-2,-1,1,2\}$ defined as

$$f\left(u\_{i}u\_{i+1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \left⌈\frac{n}{2}\right⌉-1 \\2&;i=\left⌈\frac{n}{2}\right⌉ \\-1&;\left⌈\frac{n}{2}\right⌉+1\leq i\leq n-1 \end{matrix}\right.$$

$$f\left(u\_{n}u\_{1}\right)=-2.$$

**Table 17**

|  |  |  |
| --- | --- | --- |
| $$n\geq 4$$ | Edge Conditions | Vertex Conditions |
| $n$ is even | $$e\_{f}\left(1\right)=\frac{n-2}{2}=e\_{f}\left(-1\right)$$$$e\_{f}\left(2\right)=1=e\_{f}\left(-2\right)$$ | $$v\_{f^{\*}}\left(1\right)=1=v\_{f^{\*}}\left(-1\right)$$$$v\_{f^{\*}}\left(2\right)=\frac{n-4}{2}=v\_{f^{\*}}\left(-2\right)$$$$v\_{f^{\*}}\left(3\right)=1=v\_{f^{\*}}\left(-3\right)$$ |
| $n$ is odd | $$e\_{f}\left(1\right)=\frac{n-1}{2},e\_{f}\left(-1\right)=\frac{n-3}{2}$$$$e\_{f}\left(2\right)=1=e\_{f}\left(-2\right)$$ | $$v\_{f^{\*}}\left(1\right)=1=v\_{f^{\*}}\left(-1\right)$$$$v\_{f^{\*}}\left(2\right)=\frac{n-3}{2},v\_{f^{\*}}\left(-2\right)=\frac{n-5}{2}$$$$v\_{f^{\*}}\left(3\right)=1=v\_{f^{\*}}\left(-3\right)$$ |

Hence, $ C\_{n} $ is $H\_{3}$cordial.

***Illustration 2.22*** $H\_{3}$ –cordial labeling of cycle $ C\_{8} $ is shown in below Figure.



***Remarks 2.23*** Consider cycle $ C\_{n}$. As per barycentric subdivision of $ C\_{n}(n\geq 3)$ is again a path $ C\_{2n}$ which is also is $ H\_{3}$cordial as per Theorem 2.19. Hence we have the following.

***Theorem 2.24***The barycentricsubdivision of cycle$ S(C\_{n})(n\geq 2)$ is $H\_{3}$cordial.

***Theorem 2.25*** Thesuper subdivision of cycle$ SS(C\_{n})(n\geq 4)$ is $H\_{3}$cordial.

***Proof*:** Let $V(SS(C\_{n}))=\left\{u\_{i},u\_{ij};1\leq i\leq n ,1\leq j\leq m\right\}$ and $ E(SS(C\_{n}))=\left\{u\_{i}u\_{ij},u\_{ij}u\_{i+1},,u\_{1}u\_{nj};1\leq i\leq n,1\leq j\leq m\right\}$.

Consider a function $f:E(SS(C\_{n}))\rightarrow \{-2,-1,1,2\}$ defined as

**Type 1:** $ m $is even and $n $is odd.

$$f\left(u\_{i}u\_{i1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n-1}{2}\\2&;i=\frac{n+1}{2} \\-1&;Otherwise \end{matrix}\right.$$

$$f\left(u\_{i1}u\_{i+1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n-1}{2} \\-1&;\frac{n+1}{2}\leq i\leq n-1\end{matrix}\right.$$

$$f\left(u\_{1}u\_{n1}\right)=-2$$

$$f\left(u\_{1}u\_{n2}\right)=2$$

$$f\left(u\_{n}u\_{n2}\right)=1$$

$f\left(u\_{i}u\_{i2}\right)=f\left(u\_{i2}u\_{i+1}\right)=(-1)^{i+1}$ ; $1\leq i\leq n-1$

$f\left(u\_{i}u\_{ij}\right)=f\left(u\_{ij}u\_{i+1}\right)=(-1)^{j}$ ; $3\leq j\leq m.$

**Table 18**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| $m $is even and $n $is odd | $$e\_{f}\left(1\right)=mn-1,e\_{f}\left(-1\right)=mn-2$$$$e\_{f}\left(2\right)=2,e\_{f}\left(-2\right)=1$$ | $$v\_{f^{\*}}\left(1\right)=1,v\_{f^{\*}}\left(-1\right)=0$$$$v\_{f^{\*}}\left(2\right)=\frac{n\left(m+1\right)-3}{2}$$$$v\_{f^{\*}}\left(-2\right)=\frac{n\left(m+1\right)-5}{2}$$$$v\_{f^{\*}}\left(3\right)=2,v\_{f^{\*}}\left(-3\right)=1$$ |

**Type 2:** $m $ and $ n $ both are even.

$$f\left(u\_{1}u\_{n1}\right)=-2$$

$$f\left(u\_{i}u\_{i1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n}{2} \\-1&;Otherwise\end{matrix}\right.$$

$$f\left(u\_{i1}u\_{i+1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n-2}{2} \\2&;i=\frac{n}{2} \\-1&;\frac{n+2}{2}\leq i\leq n-1\end{matrix}\right.$$

$f\left(u\_{i}u\_{i2}\right)=f\left(u\_{i2}u\_{i+1}\right)=(-1)^{i+1}$ ; $1\leq i\leq n$

$f\left(u\_{i}u\_{ij}\right)=f\left(u\_{ij}u\_{i+1}\right)=(-1)^{j}$ ; $3\leq j\leq m,1\leq i\leq n.$

**Table 19**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| and $n $both are even | $$e\_{f}\left(1\right)=nm-1=e\_{f}\left(-1\right)$$$$e\_{f}\left(2\right)=1=e\_{f}\left(-2\right)$$ | $$v\_{f^{\*}}\left(1\right)=1=v\_{f^{\*}}\left(-1\right)$$$$v\_{f^{\*}}\left(2\right)=\frac{n\left(m+1\right)-4}{2}=v\_{f^{\*}}\left(-2\right)$$$$v\_{f^{\*}}\left(3\right)=1=v\_{f^{\*}}\left(-3\right)$$ |

**Type 3:** $ m $ and $n $ both are odd.

$$f\left(u\_{1}u\_{n1}\right)=-2$$

$$f\left(u\_{i}u\_{i1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n-1}{2}\\2&;i=\frac{n+1}{2} \\-1&;Otherwise \end{matrix}\right.$$

$$f\left(u\_{i1}u\_{i+1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n-1}{2} \\-1&;\frac{n+1}{2}\leq i\leq n-1\end{matrix}\right.$$

$f\left(u\_{i}u\_{ij}\right)=f\left(u\_{ij}u\_{i+1}\right)=(-1)^{j}$ ; $2\leq j\leq m ,1\leq i\leq n.$

**Type 4:** $m$ is odd and$n $is even.

$$f\left(u\_{i}u\_{i1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n}{2} \\-1&;Otherwise\end{matrix}\right.$$

$$f\left(u\_{i1}u\_{i+1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n-2}{2} \\2&;i=\frac{n}{2} \\-1&;\frac{n+2}{2}\leq i\leq n-1\end{matrix}\right.$$

$$f\left(u\_{1}u\_{n1}\right)=-2$$

$f\left(u\_{i}u\_{ij}\right)=f\left(u\_{ij}u\_{i+1}\right)=(-1)^{j}$ ; $2\leq j\leq m,1\leq i\leq n.$

**Table 20**

|  |  |  |
| --- | --- | --- |
| $$n\geq 4$$ | Edge Conditions | Vertex Conditions |
| $m $is odd | $$e\_{f}\left(1\right)=nm-1=e\_{f}\left(-1\right)$$$$e\_{f}\left(2\right)=1=e\_{f}\left(-2\right)$$ | $$v\_{f^{\*}}\left(1\right)=1=v\_{f^{\*}}\left(-1\right)$$$$v\_{f^{\*}}\left(2\right)=\frac{n\left(m+1\right)-4}{2}=v\_{f^{\*}}\left(-2\right)$$$$v\_{f^{\*}}\left(3\right)=1=v\_{f^{\*}}\left(-3\right)$$ |

Hence, $ SS(C\_{n}) $ is $H\_{3}$cordial.

***Illustration 2.26***$ SS(C\_{5}) $with$m=3 $is $H\_{3} $cordial shown in Figure.



***Theorem 2.27***The $H$-super subdivision of cycle $ HSS(C\_{n})(n\geq 3)$ is $H\_{2}$ cordial.

***Proof*:** Let $V(HSS(C\_{n}))=\left\{u\_{i},u\_{ij};1\leq i\leq n ,1\leq j\leq 4\right\}$ and $E(HSS(C\_{n}))=\left\{u\_{i}u\_{i1},u\_{i3}u\_{i+1},u\_{i1}u\_{i3},u\_{i1}u\_{i2},u\_{i3}u\_{i4},u\_{1}u\_{n3};1\leq i\leq n-1\right\}$.

Consider a function $f:E(HSS(C\_{n}))\rightarrow \{-1,1\}$ defined as

**Type 1:**$ n $is odd.

$$f\left(u\_{i}u\_{i1}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n+1}{2}\\-1&;Otherwise \end{matrix}\right.$$

$$f\left(u\_{i+1}u\_{i3}\right)=f\left(u\_{i1}u\_{i3}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n-1}{2}\\-1&;Otherwise \end{matrix}\right.$$

$$f\left(u\_{1}u\_{n3}\right)=1$$

$$f\left(u\_{i1}u\_{i2}\right)=\left\{\begin{matrix}-1&;1\leq i\leq \frac{n+1}{2}\\1&;Otherwise \end{matrix}\right.$$

$$f\left(u\_{i3}u\_{i4}\right)=\left\{\begin{matrix}-1&;1\leq i\leq \frac{n-1}{2}\\1&;Otherwise \end{matrix}\right.$$

**Table 21**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
|  $ n $is odd | $$e\_{f}\left(1\right)=\frac{5n+1}{2},e\_{f}\left(-1\right)=\frac{5n-1}{2}$$ | $$v\_{f^{\*}}\left(1\right)=2n=v\_{f^{\*}}(-1)$$$$v\_{f^{\*}}\left(2\right)=\frac{n+1}{2}, v\_{f^{\*}}\left(-2\right)=\frac{n-1}{2}$$ |

**Type 2:**$ n $is even.

$$f\left(u\_{i}u\_{i1}\right)=f\left(u\_{i1}u\_{i3}\right)=f\left(u\_{i+1}u\_{i3}\right)=\left\{\begin{matrix}1&;1\leq i\leq \frac{n}{2} \\-1&;Otherwise\end{matrix}\right.$$

$$f\left(u\_{1}u\_{n3}\right)=1$$

$$f\left(u\_{i1}u\_{i2}\right)=f\left(u\_{i3}u\_{i4}\right)=\left\{\begin{matrix}-1&;1\leq i\leq \frac{n}{2} \\1&;Otherwise\end{matrix}\right.$$

**Table 22**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
|  $ n $is even | $$e\_{f}\left(1\right)=\frac{5n}{2}=e\_{f}\left(-1\right)$$ | $$v\_{f^{\*}}\left(1\right)=2n=v\_{f^{\*}}(-1)$$$$v\_{f^{\*}}\left(2\right)=\frac{n}{2}=v\_{f^{\*}}\left(-2\right)$$ |

Hence, $ HSS(C\_{n}) $ is $H\_{2}$ cordial.

***Illustration 2.28*** $H\_{2}$ cordial labeling of $ HSS(C\_{5}) $as shown in Figure.



# **III. Conclusion**

Path, star and cycle graph are basic graphs which we have proved to be $H\_{k}$ –cordial graphs. We have derived the results on these graphs by considering operations such as barycentric subdividion, super subdivision and $H$- super subdivision.

# **iv. References**

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