**Cordial Labeling of Path, Star and Cycle Graphs**

**1st Jayshree R. Joshi**

Research Scholar, C.U. Shah University,

Assistant Professor in Mathematics,

H. & H. B. Kotak Institute of Science,

Rajkot, India.

[**jrjoshi15@gmail.com**](mailto:jrjoshi15@gmail.com)

**2nd Dharamvirsinh Parmar**   
Assistant Professor in Mathematics,

Bhavan’s Sheth R. A. College of Science,  
 Gujarat University,   
Ahmedabad, India  
[**dharamvir\_21@yahoo.co.in**](mailto:dharamvir_21@yahoo.co.in)

# **Abstract**

In this paper we investigate cordial labeling of star, path, cycle and use operation such as subdivision, super subdivision and - super subdivision on it, i.e. .

***Keywords*-** cordial labeling, cordial labeling, Subdivision, Super subdivision, -Super subdivision of graphs.

# **Introduction**

In the present work we contemplate a finite graph which is connected and undirected. We refer to a dynamic survey of graph labeling by Gallian (2020) for detailed survey on graph labeling. For all other standard terminology and notations we refer to Gross and Yellen[4]. A labeling of a graph is a mapping that carries vertices, edges or both to the set of labels (usually to the positive or non-negative integers).

A graph is said to be cordial graph if there exists a mapping from edge set to such that induced mapping from vertex set to defined by , where is the set of all edges incident to vertex, satisfies the cordiality conditions and . Map is called cordial labeling of . By extending the concept a graphis cordial graph if there exists a mapping from edge set to such that the induced mapping from vertex set to defined by , where is the set of all edges incident to vertex, satisfies the cordiality conditions andfor Map is called cordial labeling of and graph is called cordial graph [5].

Barycentric subdivision of graph is denoted as, obtained by subdividing every edge of graph . [10]. The super subdivision of any graph denoted by is obtained from graph by replacing every edge of graph by complete bipartite graph (where is positive integer)[8].

# **II. Main Result**

***Theorem 2.1*** The star graph is cordial if is even.

***Proof*:** Let and , where is apex vertex.

Consider a function defined as

;.

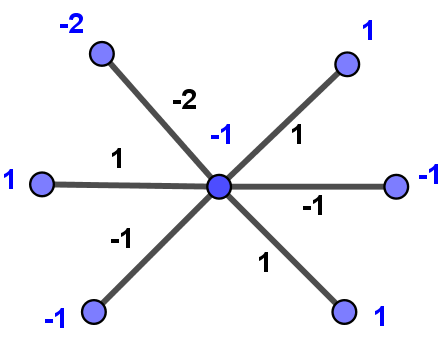
**Table 1**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is even |  |  |

For. The satisfies the condition and.

Hence, is cordial.

***Illustration 2.2***Figure shows that is cordial graph.



***Theorem 2.3*** Star graph is cordial.

***Proof*:** Let and , where is apex vertex.

**Type 1:**  is even, is cordial from Theorem 2.1. Hence it is also admits cordial labeling.

**Type 2:**  is odd.

Consider a function defined as

;

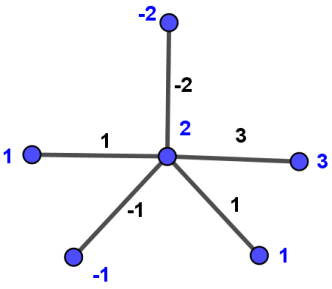
**Table 2**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is odd |  |  |

For , the graph satisfies the condition and.

Hence, is - cordial.

***Illustration 2.4***is cordial as shown in Figure.



***Theorem 2.5*** The barycentric subdivision graph of a star ( ) is cordial if is odd.

***Proof*:** Let and.

Consider a function defined as

;

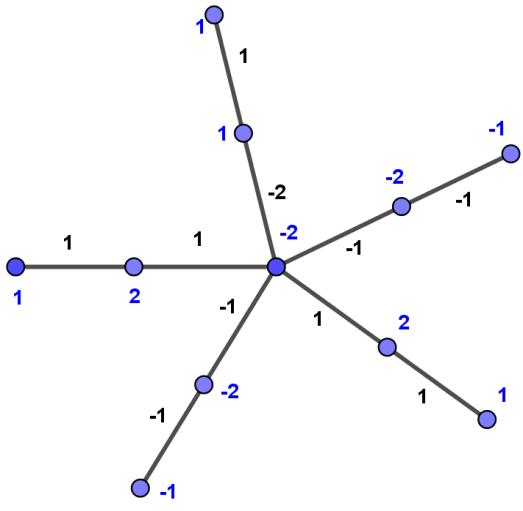
;

**Table 3**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is odd |  |  |

Hence, is cordial.

***Illustration 2.6***is cordial as shown in Figure.



***Theorem 2.7*** The barycentric subdivision graph of a star is cordial.

***Proof*:** Let and .

**Type 1:** is odd.

is cordial from Theorem 2.5, it is also admits cordial.

**Type 2:** is even.

Consider a function defined as

;

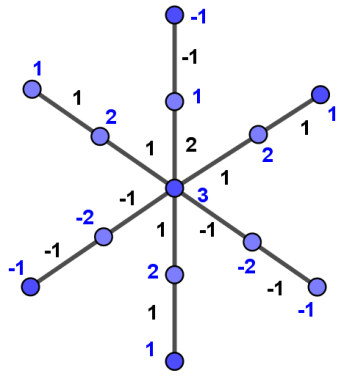
;

**Table 4**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is even |  |  |

Hence, is cordial.

***Illustration 2.8*** cordial labeling of is demonstrated in Figure.



***Theorem 2.9***Super subdivision of star graph is cordial.

***Proof*:** Let and, where is apex vertex.

Consider a function defined as

**Type 1:** is even and is odd

;

; .

**Table 5**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| even  odd |  |  |

**Type 2:** and both are even

;

;

**Table 6**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| even  even |  |  |

**Type 3:** and both are odd

;

;

**Table 7**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| odd  odd |  |  |

**Type 4:** is odd and is even

;

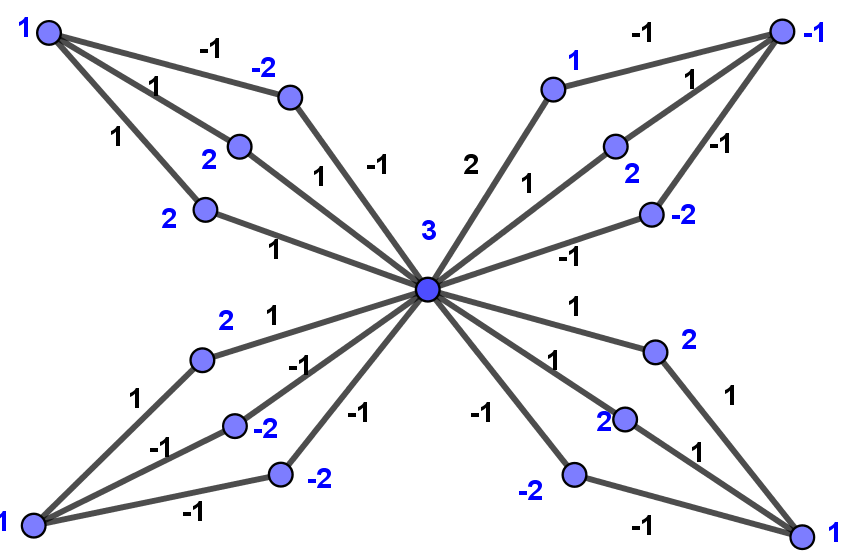
;

**Table 8**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| odd  even |  |  |

Hence, is cordial.

***Illustration 2.10***withis cordial shown in Figure.



***Theorem 2.11***The -super subdivision of path cordial.

***Proof*:** Let and , where is apex vertex.

**Type 1:** is odd, consider a function defined as

.

**Table 9**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is odd |  |  |

Hence satisfies the conditions of cordial labeling in this Type and hence the graph under consideration is cordial graph, when is odd.

**Type 2:** is even, consider a function defined as

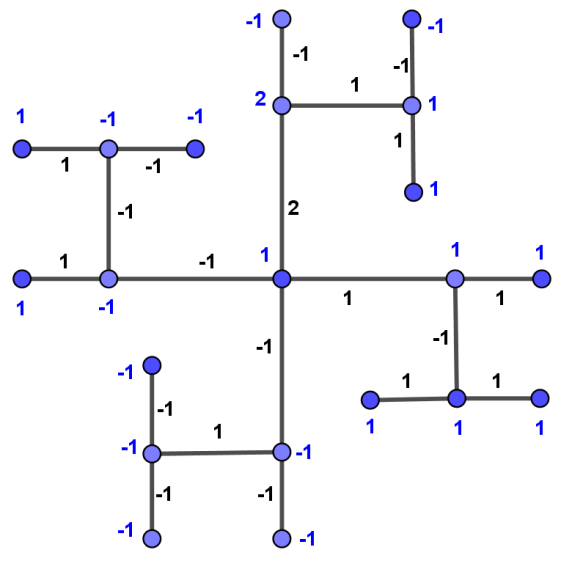
**Table 10**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is even |  |  |

In this Type satisfies the conditions of cordial labeling and hence the graph under consideration is cordial graph, when is even.

Hence, is cordial as per above Types.

***Illustration 2.12***is cordial shown in Figure.



***Theorem 2.13***Path is cordial.

***Proof*:** Let be the path .

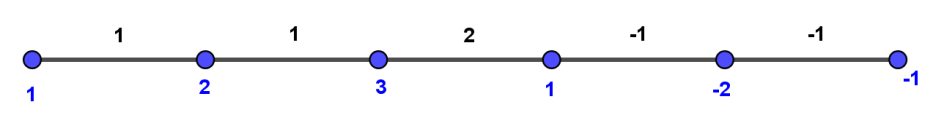
Consider a function defined as

**Table 11**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is even |  |  |
| is odd |  |  |

Hence, is cordial.

***Illustration 2.14*** cordial labeling of is as shown in below Figure.



***Remarks 2.15***

Consider path. As per barycentric subdivision of is again a path which is also is cordial as per Theorem 2.13. Hence we have the following.

***Theorem 2.16***The barycentricsubdivision of path is cordial.

***Theorem 2.17***The super subdivision of path is cordial.

***Proof*:** Let and.

Consider a function defined as

**Type 1:** is even and is odd.

;

;

**Table 12**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is even  is odd |  |  |

**Type 2:** and both are even

;

**Table 13**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is even  is even |  |  |

**Type 3:** is odd and

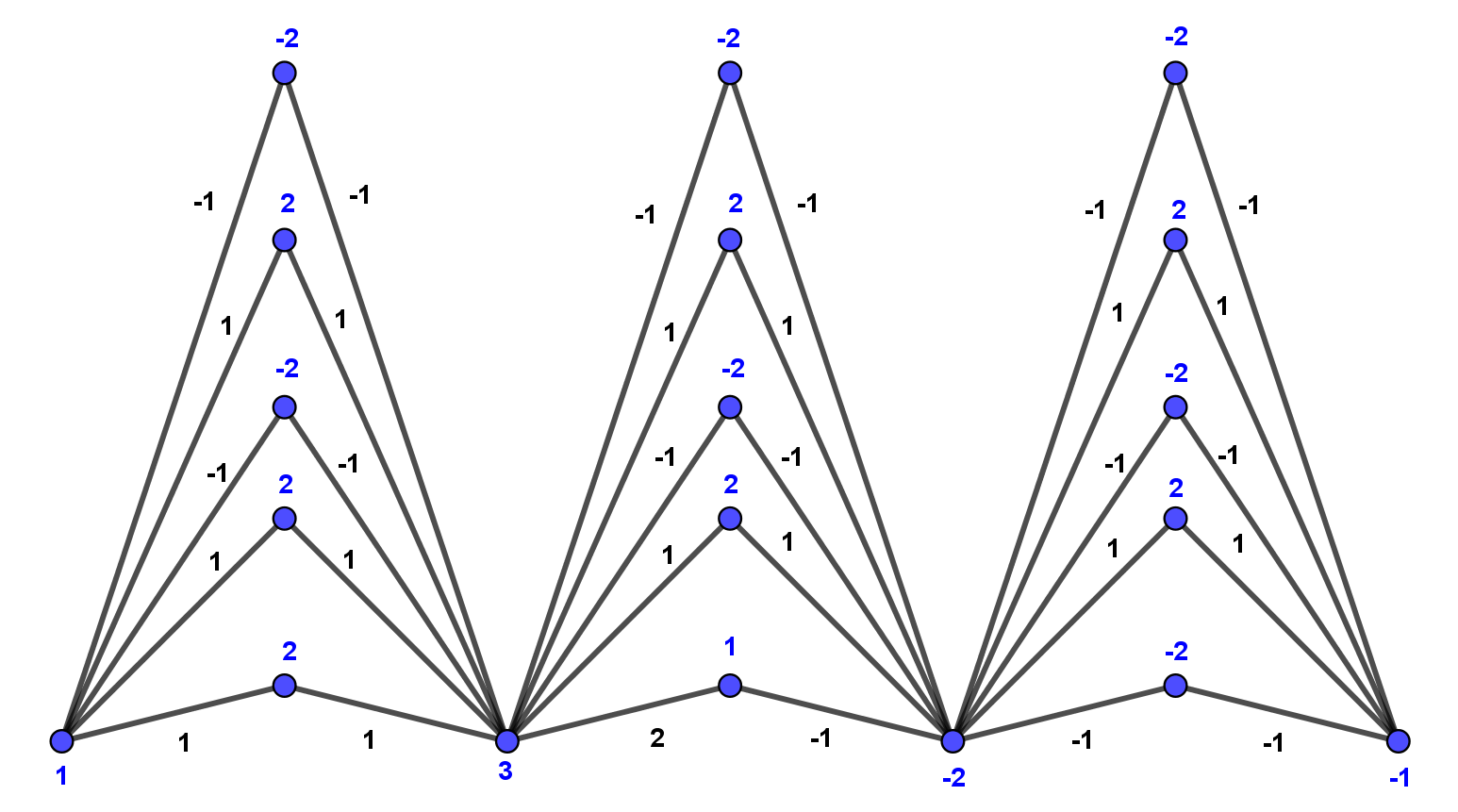
;

**Table 14**

|  |  |  |
| --- | --- | --- |
| odd | Edge Conditions | Vertex Conditions |
| is even |  |  |
| is odd |  |  |

Hence, is cordial.

***Illustration 2.18***withis cordial shown in Figure.



***Theorem 2.19***The -super subdivision of path is cordial.

***Proof*:** Let and

Consider a function defined as

**Type 1:**  is odd.

**Table 15**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is odd |  |  |

Hence satisfies the conditions cordial labeling in this Type.

**Type 2:** is even.

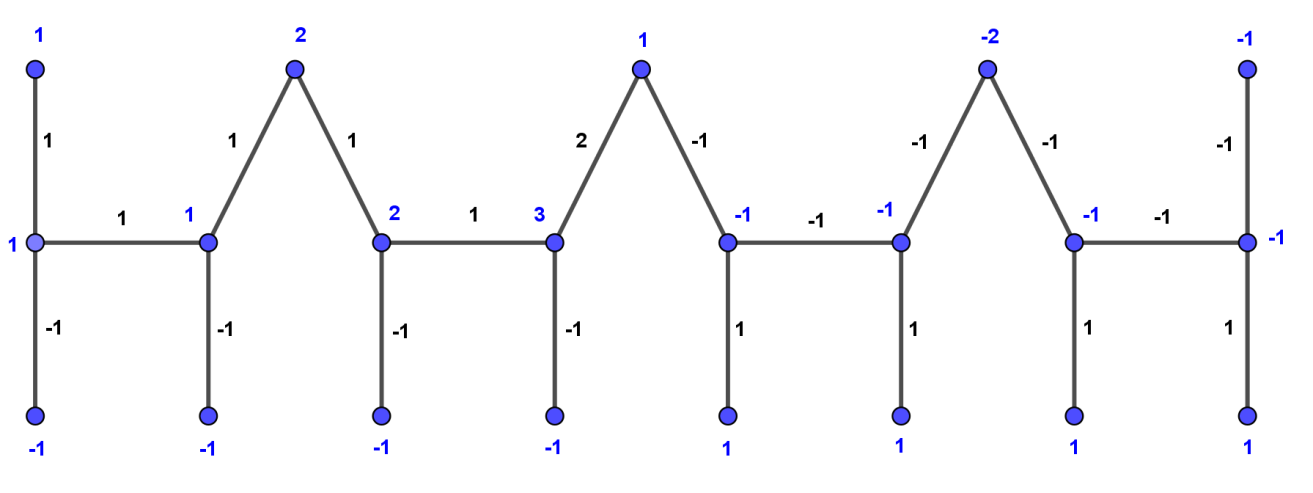
**Table 16**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is even |  |  |

Hence satisfies the conditions cordial labeling in this Type.

Hence, is cordial graph.

***Illustration 2.20*** cordial labeling of as shown in below Figure.



***Theorem 2.21***Cycle is cordial.

***Proof*:** Let and .

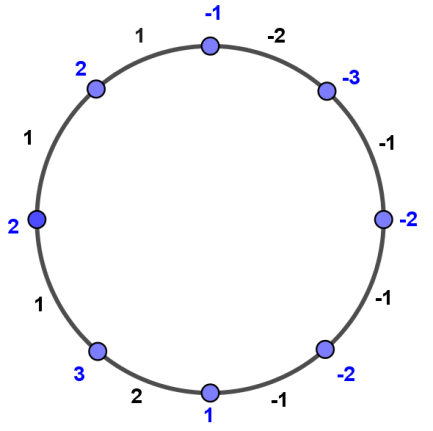
Consider a function defined as

**Table 17**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is even |  |  |
| is odd |  |  |

Hence, is cordial.

***Illustration 2.22*** –cordial labeling of cycle is shown in below Figure.



***Remarks 2.23*** Consider cycle . As per barycentric subdivision of is again a path which is also is cordial as per Theorem 2.19. Hence we have the following.

***Theorem 2.24***The barycentricsubdivision of cycle is cordial.

***Theorem 2.25*** Thesuper subdivision of cycle is cordial.

***Proof*:** Let and .

Consider a function defined as

**Type 1:** is even and is odd.

;

;

**Table 18**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is even and  is odd |  |  |

**Type 2:** and both are even.

;

;

**Table 19**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| and both are even |  |  |

**Type 3:** and both are odd.

;

**Type 4:**  is odd andis even.

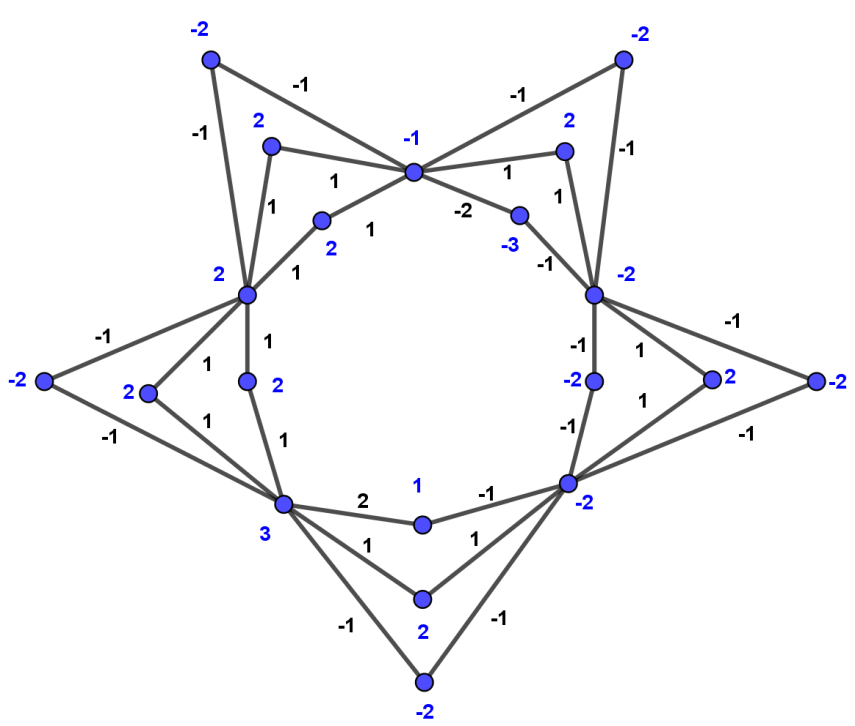
;

**Table 20**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is odd |  |  |

Hence, is cordial.

***Illustration 2.26***withis cordial shown in Figure.



***Theorem 2.27***The -super subdivision of cycle is cordial.

***Proof*:** Let and .

Consider a function defined as

**Type 1:**is odd.

**Table 21**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is odd |  |  |

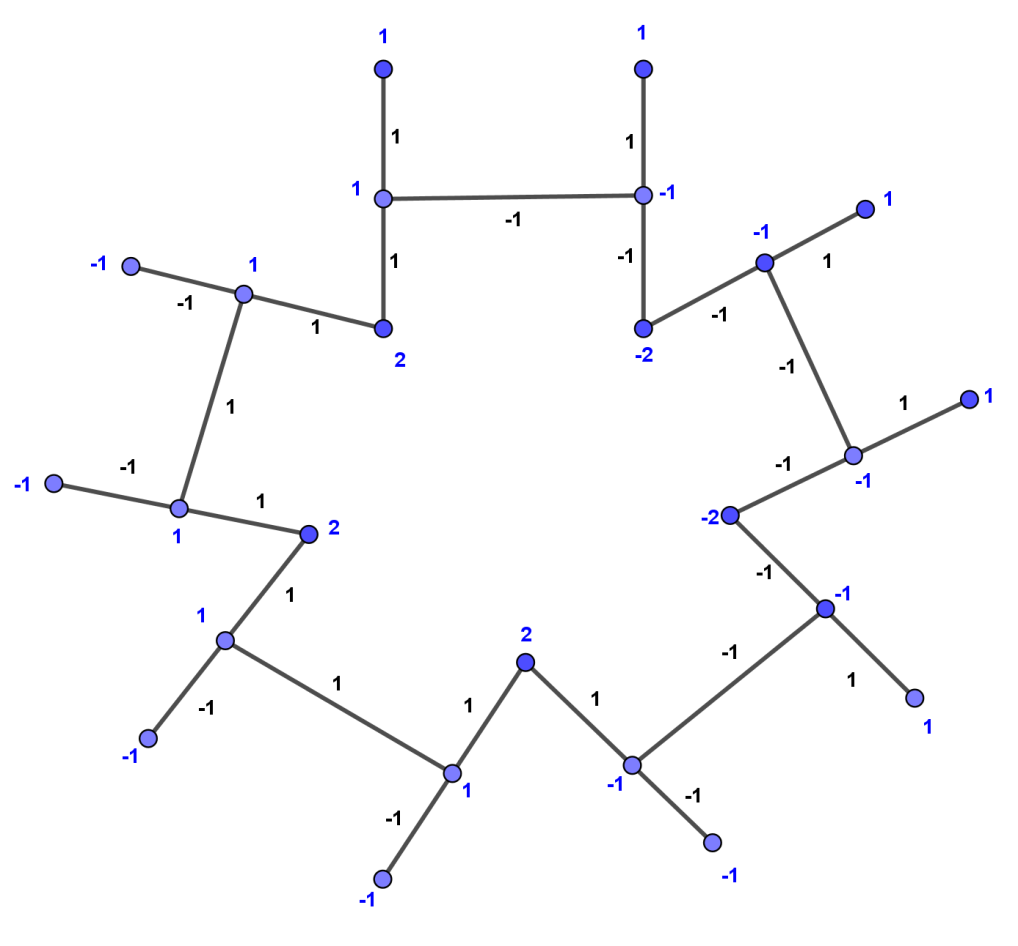
**Type 2:**is even.

**Table 22**

|  |  |  |
| --- | --- | --- |
|  | Edge Conditions | Vertex Conditions |
| is even |  |  |

Hence, is cordial.

***Illustration 2.28*** cordial labeling of as shown in Figure.



# **III. Conclusion**

Path, star and cycle graph are basic graphs which we have proved to be –cordial graphs. We have derived the results on these graphs by considering operations such as barycentric subdividion, super subdivision and - super subdivision.

# **iv. References**

1. D. Parmar and J. Joshi, “ cordial Labeling of Triangular Snake Graph”, Journal of Applied Science and Computations, vol. 6, 2019, pp. 2118-2123.
2. I. Cahit, “H–Cordial Graphs”, Bull. Inst. Combin. Appl, vol. 18, 1996, pp. 87-101.
3. J. A. Gallian, “A Dynamic Survey of Graph Labeling”, The Electronics Journal of Combinatorics, 2019 ♯𝐷𝑆6.
4. J.Gross and J.Yellen, “*Graph Theory and its applications*”, CRC Press
5. J. R. Joshi and D. Parmar, “ - Cordial Labeling of -Polygonal Snake Graphs”, Alochana Chakra Journal, vol. 9, no. 4, pp.1924 - 1938, April 2020.
6. J. R. Joshi and D. Parmar, “ cordial Labeling of Some Graph and its Corona Graphs”, International Journal of Aquatic Science, vol. 12, no. 2, 2021, pp. 1519 -1534.
7. M. Ghebleh and R. Khoeilar, “A note on: “Hcordial graphs”, Bull. Inst. Combin. Appl, vol. 31, 2001, pp. 60-68.
8. P. Jeyanthi and R. Gomathi, “Analytic Odd Mean Labeling of Super Subdivision and -Super Subdivision of Graphs”, Journal of Emerging Technologies and Innovatives Research, vol. 6, 2019, pp. 541-551.
9. S. Abhirami, R. Vikramaprasad, R. Dhavaseelan, “ Even Sum Cordial Labeling for some new Graphs”, International Journal of Mechanical Engineering and Technology, vol. 9, no. 2, pp. 214-220, February 2018.
10. S. K. Vaidya, K. K. Kanani, P. L. Vihol and N. A. Dani, “ Some Cordial Graphs in the Context of Barycentric Subdivision”, International Journal of Contemporary Mathematical Sciences, vol. 4, no. 30, 2009, pp. 1479-1492.