**Improved Correlation Coefficients of Fermatean Pentapartitioned single valued neutrosophic sets and interval Fermatean Pentapartitioned neutrosophic sets for multiple attribute decision making**

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**Abstract**: A correlation coefficients is one of the statistical measure which helps to find the degree of changes to the value predict change to the value of another. Fermatean Pentapartitioned single valued neutrosophic sets is an improvisation of wang’s single valued neutrosophic sets. In this paper we have studied the Improved Correlation Coefficients of Fermatean Pentapartitioned single valued neutrosophic sets and interval Fermatean Pentapartitioned neutrosophic sets and investigate its properties. Further we have applied this concept in multiple attribute decision making methods with Fermatean Pentapartitioned single valued neutrosophic environment and interval Fermatean Pentapartitioned single valued neutrosophic environment. Finally we illustrated an example in the above proposed method to the multiple attribute decision making problems.

**Keywords:** Fermatean Pentapartitioned single valued neutrosophic sets, Interval Fermatean Pentapartitioned neutrosophic sets, improved correlation coefficient.

**Introduction:** Fuzzy sets were introduced by Zadeh [21] in 1965 which allows the membership function valued in the interval [0, 1] and also it is an extension of classical set theory. As an extension of Zadhe’s fuzzy set theory intuitionistic fuzzy set(IFS) was introduced by Atanassov [1] in 1986,which consists of degree of membership and degree of non-membership and lies in the interval of [0,1]. IFS theory wirely used in the areas of logic programming, decision making problems, medical diagnosis etc.

Florentin Smarandache [11] introduce the concept of Neutrosphic set in 1995 which provides the knowledge of neutral thought by introducing the new factor called indeterminacy in the set. Therefore neutrosophic set was framed and it includes the components of truth membership function (T), indeterminacy membership function (I), and falsity membership function (F) respectively. Neutrosophic sets deals with non standard interval [0,1]. Since neutrosophic set deal the Indeterminacy effectively it plays an vital role in many application areas include information technology, decision support system, relational databse systems, medical diagonosis, multicriteria decision making problems etc.

Wang[12](2010) introduced the concept of single valued nuetrosophic sets (SVNS) which is also known as an extension of intuitionistic fuzzy sets and it became a very new hot research topic now. Rajashi Chatterjee.,et al [10] proposed the concept of Fermatean Pentapartitioned single valued neutrosophic sets which is based on Belnap’s four logic and Smarandache’s four numerical valued logic. In (FPSVNS) indeterminacy is splitted into two functions known as ‘Contradiction’ (both true and false) and ‘unknown’ (neither true nor false) so that (FPSVNS) has five components TA , CA , KA , UA , FA which also lies in the non standard unit interval [0,1].

Correlation coefficient is an effective mathematical tool to measure the strength of the relationship between two variables. In 1999 D.A Chiang and N.P. Lin [3] proposed the correlation of fuzzy sets under fuzzy environment. Correlation coefficients plays an important role in many real world problems like multiple attribute group decision making, clustering analysis, pattern recognition, medical diagnosis etc., Jun Ye [20] defined the improved correlation coefficients of single valued neutrosophic sets and interval nuetrosophic sets for multiple attribute decision making to overcome the drawbacks of the correlation of single valued neutrosophic sets (SVNSs) which is defined in [16].

In this paper Section 2 gives some basic definitions of Quadripartitioned single valued neutrosophic sets and Fermatean Pentapartitioned nuetrosophic sets and its complement, union, intersection, interval neutrosophic sets, correlation coefficient of FPSVNS. In Section 3, we introduced the concept of improved correlation coefficient of FPSVNS to overcome the drawbacks of correlation coefficient which is defined and also discussed some of its properties and decision making method using the improved correlation coefficient of FPSVNSs. In Section 4, we introduced the concept of interval Fermatean Pentapartitioned Neutrosophic sets (IFPNS) with some basic definitions and defined correlation coefficient of IFPNS. Further we have also discussed some of its properties and decision making method using the improved correlation coefficient with interval Fermatean Pentapartitioned partitioned neutrosophic environment. Section 5 an illustrative example is given in above proposed correlation method particularly in multiple criteria decision making problems. Section 6 concludes the paper.

**2. Preliminaries:**

**2.1 Quadripartitioned single valued neutrosophic sets:**

**Definition 2.1. [5]**

Neutrosophic set is defined over the non-standard unit interval [0 , 1] whereas single valued neutrosophic set is defined over standard unit interval [0,1]. It means a single valued neutrosophic set A is defined by x X}

where such that

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**Definition 2.2. [4]**

Let X be a non-empty set. A quadripartitioned single valued neutrosophic set (QSVNS) A over X characterizes each element in X by a truth membership function , a contradiction membership function , an ignorance membership function and a falsity membership function such that x X, and when X is discrete. A is represented as A = .

Definition 2.2. [15]

Consider X a universe. An object of the form A Fermatean pentapartitioned neutrosophic set (FPN) A on X A = {< x, TA , CA , KA , UA , FA ,) >: x X }

(TA)3 + (CA)3 + (KA)3+ (UA)3 + (FA)3 ≤ 3

Here, TA(x ) is the truth membership.

CA(x) is contradiction membership,

KA(x) is ignorance membership

UA (x) is unknown membership,

FA(x) is the false membership,

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**3. Fermatean pentapartioned single valued neutrosophic sets**

**3.1 Definition:**

Let X be a non-empty set. A Fermatean pentapartitioned single valued neutrosophic set (QSVNS) A over X characterizes each element in X by a truth membership function , a contradiction membership function , an ignorance membership function KA(x), a unknown membership function and a falsity membership function such that x X, and

When X is discrete. A is represented as

A =.

**3.2 Definition**

The complement of a FPSVNS is denoted by and is defined as,

**3.3 Definition**

The union of two FPSVNS A and B is denoted by and is defined as

**3.4 Definition;**

The intersection of two FPSVNS A and B and is defined as,

**3.5 Definition:**

Let X be a space of points (object) with generic elements in X denoted by x. An INS interval neutrosophic set A in X is characterized by a truth membership function, an indeterminacy membership function, and a falsity function For each point x in X, there are,

and

. Thus, an INS A can be expressed as

x X}

=

Then the sum of satisfies the condition.

. Obviously, when the upper and lower ends of the interval values of in an INS are equal, the INS reduce to the SVNS. However, SVNSs and INSs are all the subclasses of neutrosophic sets.

**3.6. Definition**

The complement of an INS A is denoted by and is defined as

, and for any x in X.

**3.7.Definition**

An INS A is contained in the other INS B, AB if and only if and .

**3.8. Definition**

Two INSs A and B are equal, written as A = B, if and only if AB and B.

**3.8.Definition: Correlation coefficient of QSVNSs**

Rajashi Chatterjee [4] defined the concept of the correlation coefficient of QSVNSs which is based on the correlation coefficient of SVNSs and is defined as follows:

K (A, B) =

--------- (1)

The correlation coefficient K (A, B) satisfies the following properties.

1. K(A,B) = K(B,A);
2. 0
3. K (A, B) = 1, iff A = B.

There will be some drawbacks in using Equation (1) which is given below.

For any two QSVNSs A and B, if and /or

for any in X (i=1,2,3,…n).

Equation (1) is undefined or unmeaningful. In this case it is not possible to use the formula which is given in Equation (1).

Equation (1) satisfies only the necessary condition of the property (3), but not the sufficient condition. That is AB. Equation (1) may be equal to 1.

**3.9. Example**

Let A and B be QSVNSs in X which are given by and

. Here obviously AB. Then

K (A, B) = = 1\_\_\_\_\_\_\_\_\_\_\_ (2)

Hence in this case it is not possible to apply in real life example problems. To overcome these type of disadvantages we shall define an improved correlation coefficient in the following section.

**4. Improved Correlation coefficients**

Based on the concept or correlation coefficient of FPSVNSs, we have defined the improved correlation coefficient of FPSVNSs in the following subsection.

**4.1. Definition** Let A and B be any two FPSVNSs in the universe of discourse

X = { then the improved correlation coefficient between A and B is defined as follows:

M (A, B) = …..(3)

Where ,

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For any and i =1, 2, 3….n.

**Theorem 4.2**

For any two FPSVNSs A and B in the universe of discourse X = {, the improved correlation coefficient M (A, B) satisfies the following properties.

1. M(A,B) = M(B,A);
2. ;
3. .

PROOF:

1. It is obvious ad straight forward.
2. Here ,,,,

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. Therefore the following inequation satisfies

Hence we have .

1. If M (A, B) = 1, then we get, =5. Since,,,

,, there are =1. And also since ,,,,,,,,,.

We get and. This implies, ,

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Hence,, , , ,

for any , and i = 1,2,3,….n.

Hence A = B. ,

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Now assume that A=B , implies , , , , for any , and i = 1,2,3,….n.

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,. Hence we get M (A, B) = 1.

The improved correlation coefficient formula which is defined in (3) is correct and also satisfy the three properties in Theorem 3.1 when we use any constant in the following expressions.

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When AB, consider the same example 2.12 we can get M (A, B) = 0.912 by applying Equation (3).

**Example 4.3.**

Let and be two FPSVNSs in X. Then obviously equation (1) is undefined. Therefore by using equation (3) we get M (A, B) = 0.912. It shows that the above defined improved correlation coefficient overcome the disadvantage of the correlation coefficient in [10]

In the following, we define a weighted correlation coefficient between FPSVNSs since the differences in the elements are considered into an account.

Let be the weight for each element in X (i=1, 2, 3, …. n), and , then the weight correlation coefficient between the FPSVNSs A and B

..(4)

If , then equation (4) reduces to equation (3). also satisfies the three properties in Theorem 3.1.

**Theorem 4.4**

Let be the weight for each element in X (i=1,2,3,…n), and , then the weight correlation coefficient between the FPSVNSs A and B which is denoted by, defined by (4) also satisfies the following properties.

1. ;
2. ;
3. it is similar to prove the properties in Theorem 3.1.

**4.5**. **Decision making method using the improved correlation coefficient of FPSVNSs**.

Multiple criteria decision making (MCDM) problems refers to make decisions when several attributes are involved in real – life problem. For example one may buy a car by analyzing the attributes which is given in terms of price, style, safety, comfort etc.

Here we consider a multiple attribute decision making problem with Feramatean pentapartitioned single valued neutrosophic information, and the characteristic of an alternative on an attribute is represented by the following FPSVNS.

….. (5)

Where and

, for and

i =1, 2, 3….m.

To make it convenient, we are considering the following five functions in terms of a fermatean pentapartitioned single valued neutrosophic value (FPSVNV).

Here the values of are usually derived from the evaluation of an alternative with respect to a criterion by the expert or decision maker. Therefore we got a fermatean pentapartitioned single valued neutrosophic decision matrix D =.

In the case of ideal alternative an ideal FPSVNV can be defined by

In the decision making method.

Hence the weight correlation coefficient between an alternative and the ideal alternative is given by,

..(6)

Where ,

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for I = 1,2,….m and j = 1,2,…n

By using the above weighted correlation coefficient (i=1, 2…m), we can derive the ranking order of all alternatives and we can choose the best one among those.

1. **Interval Fermatean Pentapartitioned Neutrosophic sets (IFPNS)**

**Definition 5.1**

An IFPNS A in x is denoted by a truth membership function, an contradiction membership function, an ignorance membership function , an unknown membership function and a falsity membership function. For each point x in X, there are

and

. Therefore an IFPNS a can be denote as

x X}

= / x X}

Then the sum of satisfies the condition,

. If the lower and upper ends of the interval values of in an IFPNS are equal then IFPNS reduces to the FPSVNS. Both IFPNS and FPSVNS are all the subclasses of Fermatean pentapartitioned neutrosophic sets (FPNS).

**Definition 5.2** The complement of an IFPNS A is denoted by and is defined as

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for any x in X.

**Definition 5.3.** An IFPNS A is contained in the other IFPNS B, iff

For any x in X.

**Definition 5.4**

Two IFPNS A and B are equal i.e., A =B, iff and .

5.5. **Correlation coefficient between IFPNSs**. In this section we proposed a correlation coefficient between IFPNS as a generalization of the improved correlation coefficient of FPSVNSs.

**Definition 5.6.** The correlation coefficient between two IFPNS A and B in the universe of discourse

is defined as follows:

N (A, B) = …..(7)

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Here we introduce a weighted a correlation coefficient between IFPNSs A and B by consider the weight of the element (I = 1,2,…n) into an account for any and I = 1,2,…n.

Let be the weight for each element (i=1, 2…n), and, then the weighted correlation coefficient between the IFPNSs A and B which is denoted by defined

In following equation (8).

…….(8)

If , then equation (8) reduces to equation (7). When

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, ,

in the IFPNS A and , , ,

, in the IFPNS B for any in X and i=1,2,…..n, then the IFPNS A and B reduces to the FPSVNSs A and B respectively, and also the equation (7) and (8) reduce to equations (3) and (4). Both N (A, B) and also satisfies the three properties of theorem 3.1 and theorem 3.2.

**Theorem 5.7**. For any two IFPNSs A and B in the universe of discourse , the correlation coefficient N (A, B) satisfies the following properties

1. N(A,B) = N(B,A);
2. ;
3. .

It is similar to prove the properties in Theorem 3.1.

**Theorem 5.8**

Let be the weight for each element and , then the weighted correlation coefficient between the IFPNSs A and B which is denoted by defined in equation (8) also satisfies the following properties.

1. ;
2. ;

It is similar to prove the properties in Theorem 3.1.

**5.9. Decision making method using the improved correlation coefficient of IFPNSs.**

Here we consider a multiple attribute decision making problem with interval Fermatean Pentapartieioned neutrosophic information, and the characteristic of an alternative on an attribute is represented by the following IFPNS.

Where and

for and I = 1,2,….m.

To make it convenient, we are considering the following five functions

, ,

, ,

, in terms of a interval fermatean pentapartitioned neutrosophic value (IFPNV)

Here the values of are usually derived from the evaluation of an alternative with respect to a criterion by the expert or decision maker. Therefore we got an interval Fermatean pentapartitioned neutrosophic decision maker .

In this an ideal IFPNV can be defined by

In the ideal alternative, Hence by applying equation (8) the weighted correlation coefficient between an alternative and the ideal alternative is given by,

= …..(9)

Where , , ,

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For i =1, 2…m and j = 1, 2, 3….n.

By using the above weighted correlation coefficient, we can derive the ranking order of all alternatives and we can choose the best one among those.

1. **Illustrative example**

This section deals the example for the multiple attribute decision making problem with the given alternative corresponds to the criteria allotted under fermatean pentapartitioned single valued neutrosophic environment and interval feramatean pentapartitioned neutrosophic environment.

* 1. **Decision making under feramatean pentapartitioned single valued neutrosophic environment**.

The example which will discuss here is about the best mobile phone among all available alternatives based on various criteria. The alternatives respectively denotes the mobile1, mobile2, mobile3. The customer must take a decision according to the following four attributes that is (1) is the cost (2) is the average space (3) is the camera quality (1) is the looks. According to this attributes we will derive the ranking order of all altenatives and based on this order customer will select the best one.

The weight vector of the above attributes is given by . Here the alternatives are to be evaluated under the above five attributes by the form of FPSVNSs. In general the evaluation of an alternative with respect to an attribute

Will be done by the questionnaire of a domain expert. In particularly, while asking the opinion about an alternative with respect to an attribute , the possibility he (or) she say that the statement true is 0.5 the statement both true and false is 0.4, the statement neither true nor false is 0.3 and the statement false is 0.2. It can be denoted in neutrosophic notation as. Continuing this procedure for all three alternatives with respect to four attributes we will get the following fermatean pentapartitioned single valued neutrosophic decision matrix.

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| / |  |  |  |  |
|  | [ | [ | [ | [ |
|  | [ | [ | [ | [ |
|  | [ | [ | [ | [ |

Then by using the proposed method we will obtain the most desirable alternative. We can get the values of the correlation coefficient by using equation (6).

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Therefore the ranking order is. The alternative (Mobile 1) is the best choice among all the three alternatives.

**6.3. Decision making under interval Fermatean pentapartitioned nuetrosophic environment.**

Consider the same example here the three possible alternatives are to be evaluated under the above four attributes by the form of IFPNSs. In general the evaluation of an alternative with respect to an attribute (i=1,2,3;j=1,2,3,4) will be done by the fermatean pentapartitioned of a domain expert. Therefore we get the following interval fermatean pentapartitioned nuetrosophic decision matrix R.

Then by using the proposed method we will obtain the most desirable alternative. We can get the values of the correlation coefficient by using equation (9).

Hence ,,

Therefore the ranking order is . The alternative (Mobile 2) is the best choice among all the three alternatives with respect to the given criteria under interval fermatean pentapartitioned neutrosophic environment.

**7 conclusion**

In this paper we have defined the improved correlation coefficient of FPSVNSs,IFPNSs and this sis applicable for some cases when the correlation coefficient of FPSVNSs defined in [ ] is undefined (or) unmeaningful and also studied its properties. Decision making is a process which plays a vital role in real life problems. The main process in decision making is recognizing the problem (or) opportunity and deciding to address it. Here we have discussed the decision making method using the improved correlation coefficient of FPSVNSs, IFPNSs and in particularly an illustrative example is given in multiple attribute decision making problems which involves the several alternatives based on various criteria. Hence our proposed improved correlation coefficient of FPSVNss, IFPNSs helps to identify the most suitable alternative to the customer based on the given criteria.

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