**QUADRIPARTITIONED NEUTROSOPHIC VAGUE GENERALIZED CLOSED SETS IN QUADRIPARTITIONED NEUTROSOPHIC VAGUE TOPOLOGICAL SPACES**

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**ABSTRACT:**  In this paper, we introduce the concepts of , , and with some of their properties and we prove some theorems based on , , , .

**1. INTRODUCTION**

Nowadays many real life problems includes in the field of engineering, economics deals with the concept of uncertainty, imprecise judgements, ambiguity etc.. . In these situations, we use [11] theory which was founded in 1965 by Zadeh to solve those ambiguity. which allows the elements to have a in the set and it lies in the real unit interval of [0, 1]. As an extension of , Atanassov introduced the concept of [1] which includes .. theory is utilized in the areas like , , , etc. Later on, in 1993, Gau & Beuhrer introduced the [4] .

After some time, in 2005 Smarandache presented the to solve problems contains insufficient, undefined and fickle information. In this theory, the elements in the set are allowed to have and . deals with uncertainty factor i.e, indeterminacy factor which is independent of truth and falsity values. Since is used to solve indeterminate and inconsistent information effectively, we apply in many fields like decision support system, semantic web services, new economy's growth, image processing, medical diagnosis etc., . In 2010 Wang et al., [5] developed and he defined some basic operations like , , , on .

In 1977, Belnap [2] introduced a new concept which includes a four valued logic in which any data is denoted by four parameters such as , , and . As an extension of this concept, Smarandache [10] developed four numerical valued in which is splitted into two terms namely and .

Hence a new set was introduced by Rajashi Chatterjee.,et al [9] in which we have four components in real unit interval [0,1]. Recently we have fused Vague set and Quadripartitioned Neutrosophic set and found Quadripartitioned Neutrosophic vague set[7]. In this paper, we introduce the concepts of , , and with some of their properties and we prove some theorems based on , , , .

**2. PRELIMINARIES**

**Definition 2.1[7]:** Let be the universe of discourse. A  **()**  on written as whose , , is defined as: ,,, Where, **(1)** **(2)** **(3)** **(4)** **(5)**

**Definition 2.2[7]:** A  onis a family of () in satisfying the following axioms:

(i)

(ii) for any

(iii) .

In this case the pair is called () and any in is known as () in . The complement of in is called () in .

**Definition 2.3 [7]:** The of two s and is a , written as whose , , and functions are related to those of and by

**Definition 2.4 [7]:** The intersection of two s and is a , written as whose , , and functions are related to those of and by

**Definition 2.5 [7]:** Let be an arbitrary family of s. Then

**Definition 2.6 [7]:** Let be and

be in . Then the and are defined by

(i)

(ii)

Also for any in , we have and .

It can also be shown that is and is in .

a) is in if and only if

b) is in if and only if

**3. QUADRIPARTITIONED NEUTROSOPHIC VAGUE GENERALIZED CLOSED SETS AND QUADRIPARTITIONED NEUTROSOPHIC VAGUE GENERALIZED PRE CLOSED SETS.**

**Definition 3.1:** A in a is called,

i) if and only if

ii) if and only if

**Definition 3.2:** A in a is called,

i) () if

ii) () if

iii) () if

iv) () if

v) () if

vi) () if

vii) () if

viii) () if

**Definition 3.3:** Let be a and

be a in . Then () and () of are defined by,

**Result 3.4:**  Let be a in , then

(i)

(ii)

**Definition 3.5:** Let be a and

be a in . Then and of are defined by,

(i )

(ii)

**Result 3.6:** Let

be a in , then

(i)

(ii)

**Definition 3.7:**  Let be a . A subset of is called if whenever and is a . Complement of set is called set.

**Theorem 3.8**: Every is a in .

**Proof:**

Let be a and where be in . Since is , [ since ]. Therefore . Hence is a set in .

**Theorem 3.9:**  Let and be sets in then is also set in .

**Proof**: Since and are sets in ,we get ⊆ and whenever where is in . This implies is also a subset of where is in . Then = . i.e.,. Therefore is set in .

**Theorem 3.10**: Let and be sets in then .

**Proof**: Since and are sets in , we get and whenever where is in . This implies is also a subset of where is . Since and and also we know that if then . Therefore and which implies that . Hence proved.

**Remark 3.11:** The intersection of two sets need not be a set.

**Theorem 3.12:**. Let be set in and then is set in .

**Proof**: Let where is in . Then implies . Since is , we get whenever . And also implies . Thus and so is set in .

**Theorem 3.13:**. A set is if and only if is ..

**Proof**. First assume that is then we get and so which is . Conversely assume that is . Then , i.e., implies that is . Hence proved.

**Definition 3.14**: Let be a

. A in is called

() if whenever and is a in .

**Definition 3.15**. Let be a and

be a in . Then () and () of are defined by,

(i)

(ii) .

**Result 3.16**: Let be a in , then

**Definition 3.17**: Let be a

. A in is called

set if whenever and is a . The family of all

set of a is denoted by ).

**Theorem 3.18**: Every () is a ()

but not conversely.

**Proof**. Let be a in and where be in . Since and is a in , . Hence is a set in .

**Theorem 3.19**: Every Neutrosophic ()

is a () but not conversely.

**Proof**. Let be a in and where be in . By hypothesis, and since, . Here . Therefore is a set in .

**Theorem 3.20**: Every () is a () set but not conversely.

**Proof**: Let be a set in and where be in . Since and by hypothesis, . Therefore is a set in .

**Theorem 3.21**: Every () set is a () set but not conversely.

**Proof**: Let be a in and where be a in . Since which implies . Therefore . Hence is a set in .

**4. QUADRIPARTITIONED NEUTROSOPHIC VAGUE GENERALIZED**

**CONNECTED SPACE AND QUADRIPARTITIONED NEUTROSOPHIC VAGUE**

**GENERALIZED COMPACT SPACE.**

**Definition 4.1**: Let be a

. A in is called

() set if whenever and is a in . The family of all sets of a is denoted by .

**Definition**.**4.2**: Let , be any two s. Then

1. A function is known as () if of every (respectively ) in is set (respectively ) in .

2. A function is known as if of every set (respectively ) in is set (respectively ) in .

3. A function is known as if is both and in for each in .

4. A function is known as if is both and set in for each in .

**Definition 4.3:** A is known as if no non empty is both and .

**Definition 4.4**: A is said to be () space if every set is a in .

**Definition**.**4.5**: Let be any . Then is known as () if there exists a and set such that and . is known as if it is not .

**Proposition 4.6:** Every space is . But the converse is not true.

**Proof**: Let be a space and assume that it is not . Hence there exist a

such that is both and in . Since every and is , respectively. It shows that is . Hence the proof.

**Theorem 4.7**: Let be a space. Then is if and only if is .

**Proof**: First assume that is . Then there exist a and set such that and . Since is space is both and . Hence is not . Conversely assume that is not . Then there exist a and in . Since every and is and , is not . Hence the proof.

**Proposition 4.8**: Let , are two s. If is surjection and is then is .

**Proof**. Let be not . Then there exists a and set in such that and . Since is , is and set in . Thus is not g-connected. Hence the proof.

**Definition 4.9:** Let be a . If a family

of sets in satisfies the condition, then it is known asof .

A finite subfamily of a of

which is also a of is known as of

.

**Definition 4.10**: A is called () if and only if every of has a .

**Theorem 4.11**: Let, be two s and be . If is then is also .

**Proof:** Let be a in with

Since is ,

is of .

Now, ...................... (1)

Since is , there exists a finite sub cover such that,

Hence,

[by (1)]

Therefore is .

**Definition 4.12**: Let be a and be a in . If a family

of sets in satisfies the condition

then it is known as cover of .

A finite subfamily of a cover

of which is also a cover of is known as of

.

**Definition 4.13**: A in is known as if and only if every of has a .

**Theorem 4.14**: Let , be any two s and be an function. If is in then is in .

**Proof.** Let be a cover of in i.e,

Since is ,

is of in .

Now,

Since is , then there exist a such that,

Hence,

is in .

**CONCLUSION:** We have introduced the concepts of , , and with some of their properties and we prove some theorems based on , , , .

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