On Some typical kind of Continuity in Soft Topological Space

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ABSTRACT

In this paper we have made an attempt to make results on some typical kind of continuous functions of Soft J Open and Soft J Closed sets in Soft Topological spaces. This study also describes the characterization of continuity with reference to our Soft J Open sets in Soft Topological Spaces.

Keywords—Soft J Open set, Soft J Continuous Functions, Soft Strongly J Continuous Functions and Soft Perfectly J Continuous Functions.

#  INTRODUCTION

 *Soft* set hypothesis was proposed by Molodtsov [4] in 1999 to manage vulnerability in a parametric way. A *Soft* set is a defined group of sets, instinctively *Soft* on the grounds that the limit of the set relies upon the boundaries. One idea of a set is the idea of dubiousness. Molodtsov [6] proposed *Soft* set as a totally nonexclusive numerical instrument for displaying vulnerabilities. There is no restricted condition to the depiction of articles. One of the critical benefits of soft topological spaces lies in their capacity to deal with complex frameworks with deficient or problematic information. They can display questionable conditions, rough thinking, and manage fractional data in a more regular and natural way contrasted with customary topological spaces.

# PRELIMINARIES

**Definition 2.1:[6]** A $Soft$ set $F\_{A}$ on the universe $X$ is deﬁned by the set of ordered pairs $F\_{A}=\{(x,f\_{a}(x)):x\in E and f\_{a}(x)\in P(X)\}$, where $f\_{a}:A\rightarrow P(X)$ such that $f\_{a}\left(x\right)=ϕ$ for all $x\notin A$. Hence $f\_{a}$ is called an $approximate function of the Soft$ set $F\_{A}$. The value of $f\_{a}$ may be $arbitrary$, some of them may be $empty$, some may have non empty intersection.

**Definition 2.2: [2]**

1. A $Soft$ set $(F,A)$ over $X$ is called as a $Null Soft$ Set denoted by $F\_{ϕ}$ or $\tilde{ϕ}$ if for all $e\in A$, $F(e)=ϕ$.
2. A $Soft$ set $(F,E)$ over $X$ is called as an $Absolute Soft$ Set denoted by $F\_{X}$ or $\tilde{X}$ if for all $e\in A$*,* $F(e)=X.$

**Definition 2.3:[6]** Suppose $τ$ be a collection of $Soft$ sets over $X$ with a fixed set $E$ of parameters. Then, $τ$ is called a $Soft topology$on $X$ if

1. $\tilde{ϕ}, \tilde{X}$ belongs to $τ$.
2. The artbitrary union of $Soft$ sets in $τ$ is again in $τ$.
3. The finite intersection $Soft$ sets in $τ$ is again in $τ$.

The term $(X,τ,E)$ is called $Soft topological Space over X$. The members of $τ$ are called $Soft open$sets in $X$ and complements of them are called $Soft closed$sets in $X$.

**Definition 2.4:[3]** A $Soft$ set $(W,E)$ of a $Soft topological space$ $(X,τ,E)$ is known as a $Soft J Closed$ **set** if $s^{\*}Cl(W,E)\tilde{⊆}Int(V,E)$ when $(W,E)\tilde{⊆}(V,E)$ and $(V,E)$ is $Soft \hat{g}-open$. $SJC(X,τ,E)$ stands for the set of all $Soft J closed sets$.

**Definition 2.5.[3]** A $Soft set$ $(Q,E)$ of a $Soft topological$ space $(X,τ,E)$ is known as a $Soft J Open$ **set** if its complement is a $Soft J closed$ set. $SJO(X,τ,E)$ stands for the set of all $Soft J open$ sets.

**Definition 2.6.** **[1,5]** A map $f:(X,τ,E)\rightarrow (Y,σ,K)$ is called a

1. $Soft continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft open$ in $(X,τ,E)$.
2. $Soft semi-continuous$if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft semi-open$ in $(X,τ,E)$.
3. $Soft pre-continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft pre-open$ in $(X,τ,E)$.
4. $Soft α-continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft α-open$ in $(X,τ,E)$.
5. $Soft β-continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft β-open$ in $(X,τ,E)$.
6. $Soft g continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft g open$ in $(X,τ,E)$.
7. $Soft sg continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft sg-open$ in $(X,τ,E)$.
8. $Soft gs continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft gs-open$ in $(X,τ,E)$.
9. $Soft gp continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft gp-open$ in $(X,τ,E)$.
10. $Soft gpr continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft pre-open$ in $(X,τ,E)$.
11. $Soft αg-continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft αg-open$ in $(X,τ,E)$.
12. $Soft gα-continuous$ if the $inverse image$ of every $Soft open $set in $(Y,σ,K)$ is $Soft gα-open$ in $(X,τ,E)$.
13. $Soft \hat{g}-continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft \hat{g}-open$ in $(X,τ,E)$.

$Soft JP continuous$ if the $inverse image$ of every $Soft open$ set in $(Y,σ,K)$ is $Soft JP open$ in $(X,τ,E)$.

**Result 2.7.[6]**

* + - 1. Each one of the $Soft semi closed$ set remains $Soft J closed$.
			2. Each one of the $Soft closed set$ remains $Soft J closed$.
			3. Each one of the $Soft α-closed$ set remains $Soft J Closed$.
			4. Each one of the $Soft open set$ remains $Soft J open$.
			5. Each one of the $Soft semi-open$ set remains $Soft J open$.
			6. Each one of the $Soft α-open$ set remains $Soft J open$.
			7. Each one of the $Soft J open$ set remains $Soft gs-open$.

# $SOFT TOTALLY J CONTINUOUS$ FUNCTIONS

**Deﬁnition 3.1:** A map $f:(X,τ,E)\rightarrow (Y,σ,K)$ is known as $Soft totally J continuous$ if the $inverse-image$ of each one of the $Soft open$ set in $(Y,σ,K)$ is both $Soft J closed$ and $Soft J open$ (i.e $Soft J clopen$) in $(X,τ,E)$.

**Theorem 3.2:** Each one of the $Soft perfectly J continuous$ map is $Soft totally J continuous$.

**Proof:** Let $f:(X,τ,E)\rightarrow (Y,σ,K)$ be $Soft perfectly J continuous$ and $(U,K)$ be a $Soft open$ set in $(Y,σ,K)$. Thereon $(U,K)$ is $Soft J open$ in $(Y,σ,K)$. Since $f$ is $Soft Perfectly J continuous$, $f^{-1}(U,K)$ is $Soft clopen$ in $(X,τ,E)$. By Result 2.7, $f$ is $Soft totally J continuous$.

**Remark 3.3:** It is observed from the subsequent illustration that the reverse implication of the above theorem is incorrect.

**Example 3.4:** Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\}.$ Define $m :X\rightarrow Y$ and $n :E\rightarrow K$ as $m\left(x\_{1}\right)=y\_{1}, m\left(x\_{2}\right)=y\_{2}$ and $n\left(e\_{1}\right)=k\_{1}, n\left(e\_{2}\right)=k\_{2}$. Consider the Soft topologies $τ=\{\tilde{ϕ}, \tilde{X},(M\_{1},E),(M\_{2},E)\}$ where $\left(M\_{1},E\right)$ and $(M\_{2},E)$ are described this way: $M\_{1}\left(e\_{1}\right)=\left\{x\_{1}\right\},M\_{1}\left(e\_{2}\right)=ϕ,M\_{2}\left(e\_{1}\right)=\left\{x\_{1}\right\},M\_{2}\left(e\_{2}\right)=\left\{x\_{2}\right\}$ and $σ=\{\tilde{ϕ}, \tilde{Y},\left(N\_{1},K\right),(N\_{2},K)\}$ where $\left(N\_{1},K\right)$ and $(N\_{2},K)$ are described this way: $N\_{1}\left(k\_{1}\right)=ϕ,N\_{1}\left(k\_{2}\right)=\left\{y\_{1}\right\}$ and $N\_{2}\left(k\_{1}\right)=\left\{y\_{1}\right\},N\_{2}\left(k\_{2}\right)=\left\{y\_{1}\right\}$. Precisely the mapping $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft totally J continuous. The $Soft$ set $\left(H,K\right)$ defined as $H\left(k\_{1}\right)=ϕ,H\left(k\_{2}\right)=\left\{y\_{1}\right\}$ is a $Soft J open$ set in $(Y,σ,K)$. But $f^{-1}\left(H,K\right)=\{\left(e\_{1},ϕ\right),\left(e\_{2},x\_{1}\right)\}$ is not $Soft clopen$ in $(X,τ,E)$. Hence $f$ is not $Soft perfectly J continuous$.

**Theorem 3.5:** Each one of the $Soft totally J continuous$ map is $Soft J continuous$.

**Proof:** Let $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft totally J continuous$ and $(A,K)$ be $Soft open$ set in $(Y,σ,K)$. Since $f$ is $Soft perfectly J continuous$, $f^{-1}(A,K)$ is $Soft clopen$ in $(X,τ,E)$. Then $f^{-1}(A,K)$ is $Soft J clopen$ in $(X,τ,E)$. Thereupon $f$ is $Soft totally J continuous$.

**Remark 3.6:** It is observed from the subsequent illustration that the reverse implication of the above theorem is incorrect.

**Example 3.7:** Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\}.$ Define $p :X\rightarrow Y$ and $q :E\rightarrow K$ as $p\left(x\_{1}\right)=y\_{1}, p\left(x\_{2}\right)=y\_{2}$ and $q\left(e\_{1}\right)=k\_{1}, q\left(e\_{2}\right)=k\_{2}$. Consider the Soft topologies $τ=\{\tilde{ϕ}, \tilde{X},(F\_{1},E),(F\_{2},E)\}$ where $\left(F\_{1},E\right)$ and $(F\_{2},E)$ are described this way: $F\_{1}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{1}\left(e\_{2}\right)=\left\{x\_{1}\right\},F\_{2}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{2}\left(e\_{2}\right)=\left\{x\_{1},x\_{2}\right\}$ and $σ=\{\tilde{ϕ}, \tilde{Y},(H,K)\}$ where $\left(H,K\right)$ is described this way: $H\left(k\_{1}\right)=ϕ,H\left(k\_{2}\right)=\left\{y\_{1},y\_{2}\right\}$. Let $g:(X,τ,E)\rightarrow (Y,σ,K)$ be a $Soft$ mapping. Precisely $g$ is $Soft J continuous$ but not $Soft totally J continuous$, because $g^{-1}\left(H,K\right)=\{\left(e\_{1},x\_{1},x\_{2}\right),(e\_{2},ϕ)\}$ is not $Soft J clopen$ in $(X,τ,E)$.

**Theorem 3.8:** Each one of the $Soft totally J continuous$ map is $Soft JA continuous$.

**Proof:** $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft J continuous$. Let $(A,K)$ be a $Soft closed$ set in $(Y,σ,K)$. Then $f^{-1}(A,K)$ is $Soft J closed$ in $(X,τ,E)$. Also, $f^{-1}(A,K)$ is $Soft JA closed$ in $(X,τ,E)$. Thus $f$ is $Soft JA continuous$.

# $SOFT CONTRA J CONTINUOUS$ FUNCTIONS

**Deﬁnition 4.1:** A map $f:(X,τ,E)\rightarrow (Y,σ,K)$ is known as $Soft Contra J continuous$if the $inverse-image$ of each one of the $Soft open$ set in $(Y,σ,K)$ is $Soft J closed$ in $(X,τ,E)$.

**Example 4.2:** Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\}.$ Define $p :X\rightarrow Y$ and $q :E\rightarrow K$ as $p\left(x\_{1}\right)=y\_{1}, p\left(x\_{2}\right)=y\_{2}$ and $q\left(e\_{1}\right)=k\_{1}, q\left(e\_{2}\right)=k\_{2}$. Consider the $Soft$ topologies $τ=\{\tilde{ϕ}, \tilde{X},(F\_{1},E),(F\_{2},E)\}$ where $\left(F\_{1},E\right)$ and $(F\_{.2},E)$ are deﬁned as $F\_{1}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{1}\left(e\_{2}\right)=\left\{x\_{1}\right\},F\_{2}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{2}\left(e\_{2}\right)=\left\{x\_{1},x\_{2}\right\}$ and $σ=\{\tilde{ϕ}, \tilde{Y},(H,K)\}$ where $\left(H,K\right)$ is described this way: $H\left(k\_{1}\right)=\left\{y\_{1},y\_{2}\right\},H\left(k\_{2}\right)=ϕ$. Let $g:(X,τ,E)\rightarrow (Y,σ,K)$ be a $Soft$ mapping. Precisely $g$ is $Soft contra J continuous$.

**Proposition 4.3:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft contra semi continuous$ then it is $Soft contra J continuous$.

**Proof:** It is verified by Result 2.7, that each one of the $Soft semi closed$ set is $Soft J closed$ in $(X,τ,E)$.

**Proposition 4.4:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft contra continuous$ then it is $Soft contra J continuous$.

**Proof:** It is verified by Result 2.7, that each one of the $Soft closed$ set is $Soft J closed$ in $(X,τ,E)$.

**Proposition 4.5:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft contra α-continuous$ then it is $Soft contra J continuous$.

**Proof:** It is proved by Result 2.7, that each one of the $Soft α-closed$ set is $Soft J closed$ in $(X,τ,E)$.

**Result 4.6:** It is observed from the subsequent illustration that the reverse implication of the above propositions 4.3, 4.4, 4.5 are incorrect.

**Example 4.7:** Consider the $Soft open$ set $(H,K)$ in Example 4.2. Here, $g^{-1}\left(H,K\right)=\{\left(e\_{1},x\_{1},x\_{2}\right),(e\_{2},ϕ)\}$ is not $Soft$ $semi-closed$ ($Soft$ closed, $Soft α-closed$) in $(Y,σ,K)$. Hence $g:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft contra J continuous$ but not $Soft contra semi continuous$ ($Soft contra continuous$, $Soft contra α-continuous$).

**Remark 4.8:** $Soft J continuity$ and $Soft contra J continuity$ are independent. It is observed from the subsequent illustration.

**Example 4.9:**

1. Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\}.$ Define $u :X\rightarrow Y$ and $v :E\rightarrow K$ as $u\left(x\_{1}\right)=y\_{1}, u\left(x\_{2}\right)=y\_{2}$ and $v\left(e\_{1}\right)=k\_{1}, v\left(e\_{2}\right)=k\_{2}$. Consider the $Soft$ topologies $τ=\{\tilde{ϕ}, \tilde{X},\left(F\_{1},E\right),\left(F\_{2},E\right)\}$ where $\left(F\_{1},E\right)$ and $(F\_{2},E)$ are described this way: $F\_{1}\left(e\_{1}\right)=x\_{2},F\_{1}\left(e\_{2}\right)=\left\{x\_{2}\right\},F\_{2}\left(e\_{1}\right)=\left\{x\_{1},x\_{2}\right\},F\_{2}\left(e\_{2}\right)=\left\{x\_{2}\right\}$ and $σ=\{\tilde{ϕ}, \tilde{Y},(H\_{1},K),(H\_{2},K),(H\_{3},K)\}$ where $(H\_{1},K),(H\_{2},K)$ and $(H\_{3},K)$ are described this way: $H\_{1}\left(k\_{1}\right)=\left\{y\_{1}\right\},H\_{1}\left(k\_{2}\right)=ϕ,H\_{2}\left(k\_{1}\right)=ϕ,H\_{2}\left(k\_{2}\right)=\left\{y\_{2}\right\},H\_{3}\left(k\_{1}\right)=\left\{y\_{1}\right\},H\_{3}\left(k\_{2}\right)=\left\{y\_{2}\right\}$. Precisely the mapping $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft J continuous$. The inverse-image of the $Soft open$ set $(H\_{2},K)$, $f^{-1}(H\_{2},K)=\{\left(e\_{1},ϕ\right),\left(e\_{2},x\_{2}\right)\}$ is not a $Soft J closed$ set in $(X,τ,E)$. Hence $f$ is not $Soft contra J continuous$.
2. Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\}.$ Define $p :X\rightarrow Y$ and $q :E\rightarrow K$ as $p\left(x\_{1}\right)=y\_{1}, p\left(x\_{2}\right)=y\_{2}$ and $q\left(e\_{1}\right)=k\_{1}, q\left(e\_{2}\right)=k\_{2}$. Consider the $Soft topologies$ $τ=\{\tilde{ϕ}, \tilde{X},\left(M\_{1},E\right),\left(M\_{2},E\right)\}$ where $\left(M\_{1},E\right)$ and $(M\_{2},E)$ are deﬁned as $M\_{1}\left(e\_{1}\right)=x\_{2},M\_{1}\left(e\_{2}\right)=\left\{x\_{2}\right\},M\_{2}\left(e\_{1}\right)=\left\{x\_{1},x\_{2}\right\},M\_{2}\left(e\_{2}\right)=\left\{x\_{2}\right\}$ and $σ=\{\tilde{ϕ}, \tilde{Y},(H,K)\}$ where $\left(H,K\right)$ is described this way: $H\left(k\_{1}\right)=\left\{y\_{1},y\_{2}\right\},H\left(k\_{2}\right)=y\_{1}$. Let $g:(X,τ,E)\rightarrow (Y,σ,K)$ be a $Soft$ mapping. Precisely $g$ is $Soft contra J continuous$. The $Soft$ set $(R,K)$ defined as $R\left(k\_{1}\right)=ϕ,R\left(k\_{2}\right)=y\_{2}$ is $Soft closed$ in $\left(Y,σ,K\right)$ but its $inverse-image$ $g^{-1}\left(R,K\right)=\{\left(e\_{1},ϕ\right),\left(e\_{2},x\_{2}\right)\}$ is not $Soft J closed$ in $(X,τ,E)$. Hence $g$ is not $Soft J continuous$.

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**Lemma 4.10:** The following properties hold for the $Soft$ subsets $(A,E), (B,E)$ of a space $(X,τ,E)$.

1. $(A,E)\tilde{⊂}SKer(A,E)$ and $(A,E)=SKer(A,E)$ if $(A,E)$ is $Soft open$ in $(X,τ,E)$.

2. $(A,E)\tilde{⊂}(B,E)$ then $SKer(A,E)\tilde{⊂}SKer(B,E)$.

**Proof:** The proof is obvious.

**Theorem 4.11:** For a $Soft$ mapping $f:(X,τ,E)\rightarrow (Y,σ,K)$ the subsequent properties are equivalent. Assume that $SJO(X,τ,E)$ is closed under any union and $SJC(X,τ,E)$ is closed under any intersection.

1. $f$ is $Soft contra J continuous$.

2. The $inverse-image$ of a $Soft closed$ set $(B,K)$ of $(Y,σ,K)$ is $Soft J open$.

3. $f(SJCl(A,E))\tilde{⊂}SKer f((A,E))$ for each one of the $Soft$ subset of $(A,E)$ of $(X,τ,E)$.

4. $SJCl(f^{-1}(C,K))\tilde{⊂}f^{-1}(SKer(C,K))$ for each one of the subset $(C,K)$ of $(Y,σ,K)$.

**Proof:**

$(1)⇒(2):$ It is evident.

$(2)⇒(3):$ Let $(A,E)$ be any $Soft$ subset of $(X,τ,E)$. Suppose $(y,e)\tilde{\notin }SKer (f(A,E))$. Then by lemma 4.10, there exists $(C,K)$ such that $f(A,E)\tilde{∩}(C,K)=\tilde{ϕ}$. Thus $(A,E)\tilde{∩}f^{-1}(C,K)=\tilde{ϕ}$ and $SJCl(A,E)\tilde{∩}f^{-1}(C,K)=\tilde{ϕ}$. Therefore, $f(SJCl(A,E))\tilde{∩}(C,K)=\tilde{ϕ}$ and $(y,e)\tilde{\notin }f(SJCl(A,E)$. Thereon $f(SJCl(A,E))\tilde{⊂}SKer f((A,E))$ for every $Soft$ subset of $(A,E)$ of $(X,τ,E)$.

$(3)⇒(4):$ Let $(C,K)$ be any $Soft$ subset of $(Y,σ,K)$. Then by (3) and Lemma 4.10, $f(SJCl(f^{-1}(C,K)))\tilde{⊂}SKer(f(f^{-1}(C,K)))\tilde{⊂}SKer(C,K)$ and then $SJCl(f^{-1}(C,K))\tilde{⊂}f^{-1}(SKer(C,K))$.

$(4)⇒(1):$ Let $(V,K)$ be any $Soft open$ subset of $(Y,σ,K)$. Therefore, by hypothesis and by Lemma 4.10, $SJCl(f^{-1}(V,K))\tilde{⊂}f^{-1}(SKer(V,K))= f^{-1}(V,K)$. So, $SJCl(f^{-1}(V,K)=f^{-1}(V,K)$. This reveals that $f^{-1}(V,K)$ is $Soft J closed$ in $(X,τ,E)$. Hence $f$ is $Soft contra J continuous$.

**Result 4.12:** The composition of two $Soft contra J continuous$ functions need not be $Soft contra J continuous$ and it is observed from the subsequent illustration.

**Example 4.13:** Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},Z= \left\{z\_{1},z\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\} $and$ R= \left\{r\_{1},r\_{2}\right\}.$ Define $s :X\rightarrow Y,t :Y\rightarrow Z$ and $p:E\rightarrow K,q :K\rightarrow R$ as $s\left(x\_{1}\right)=y\_{1}, s\left(x\_{2}\right)=y\_{2},t\left(y\_{1}\right)=z\_{1}, t\left(y\_{2}\right)=z\_{2}$ and $p\left(e\_{1}\right)=k\_{1}, p\left(e\_{2}\right)=k\_{2},q\left(k\_{1}\right)=r\_{1}, q\left(k\_{2}\right)=r\_{2}$. Consider the $Soft$ topologies $τ=\{\tilde{ϕ}, \tilde{X},(F\_{1},E),(F\_{2},E)\}$ where $\left(F\_{1},E\right)$ and $(F\_{2},E)$ are described this way: $F\_{1}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{1}\left(e\_{2}\right)=\left\{x\_{1}\right\},F\_{2}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{2}\left(e\_{2}\right)=\left\{x\_{1},x\_{2}\right\}$, $σ=\{\tilde{ϕ}, \tilde{Y},(H,K)\}$ where $\left(H,K\right)$ is described this way: $H\left(k\_{1}\right)=\left\{y\_{1},y\_{2}\right\},H\left(k\_{2}\right)=ϕ$ and $η=\{\tilde{ϕ}, \tilde{Z},\left(M\_{1},R\right),\left(M\_{2},R\right)\}$ where $\left(M\_{1},R\right)$ and $(M\_{2},R)$ are described this way: $M\_{1}\left(r\_{1}\right)=z\_{2},M\_{1}\left(r\_{2}\right)=\left\{z\_{2}\right\},M\_{2}\left(r\_{1}\right)=\left\{z\_{1},z\_{2}\right\},M\_{2}\left(r\_{2}\right)=\left\{z\_{2}\right\}$. Let $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be two $Soft$ mappings. Precisely $f$ and $g$ are $Soft contra J continuous$ but their composition is not $Soft contra J continuous$ because $\left(g∘f\right)^{-1}\left(H,R\right)=\{(e\_{1},x\_{2}),(e\_{2},x\_{2})\}$ is not $Soft J closed$ in $(X,τ,E)$.

**Proposition 4.14:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft J irresolute$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is $Soft contra J continuous$ then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft contra J continuous$.

**Proof:** Let $(Q,R)$ be a $Soft closed$ set in $(Z,η,R)$. Since $g$ is $Soft contra J continuous$ $g^{-1}(Q,R)$ is $Soft J open$ in $(Y,σ,K)$. Because $f$ is $Soft J irresolute$, $f^{-1}(g^{-1}(Q,R))=(g∘f)^{-1}(Q,R)$ is $Soft J open$ in $(X,τ,E)$. So $g∘f$ is $Soft contra J continuous$.

**Proposition 4.15:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft contra J continuous$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is $Soft continuous$ then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft contra J continuous$.

**Proof:** Let $(P,R)$ be a $Soft closed$ set in $(Z,η,R)$. Since $g$ is $Soft continuous$ $g^{-1}(P,R)$ is $Soft closed$ in $(Y,σ,K)$. Since $f$ is $Soft contra J continuous$ $f^{-1}(g^{-1}(P,R))=(g∘f)^{-1}(P,R)$ is $Soft J open$ in $(X,τ,E)$. Thus $g∘f$ is $Soft contra J continuous$.

**Proposition 4.16:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft contra semi continuous$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is $Soft continuous$ then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft contra J continuous$.

**Proof:** Let $(F,R)$ be a $Soft closed$ set in $(Z,η,R)$. Since $g$ is $Soft continuous$ $g^{-1}(F,R)$ is $Soft closed$ in $(Y,σ,K)$. Since $f$ is $Soft contra semi continuous$ $f^{-1}(g^{-1}(F,R))=(g∘f)^{-1}(F,R)$ is $Soft semi open$ in $(X,τ,E)$. By Result 2.7, $(g∘f)^{-1}(F,R)$ is $Soft J open$ in $(X,τ,E)$. Thus $g∘f$ is $Soft contra J continuous$.

**Proposition 4.17:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft contra α-continuous$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is $Soft continuous$ then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft contra J continuous$.

**Proof:** Let $(S,R)$ be a $Soft closed$ set in $(Z,η,R)$. Since $g$ is $Soft continuous$ $g^{-1}(P,R)$ is $Soft closed$ in $(Y,σ,K)$. Since $f$ is $Soft contra α-continuous$, $f^{-1}(g^{-1}(P,R))=(g∘f)^{-1}(P,R)$ is $Soft α-open$ in $(X,τ,E)$. By Result 2.7, $(g∘f)^{-1}(P,R)$ is $Soft J open$ in $(X,τ,E)$. Thus $g∘f$ is $Soft contra J continuous$.

**Theorem 4.18:** Let $\{(X\_{i},τ,E)∕i\in I\}$ be any family of $Soft topological spaces$. If $f:(X,τ,E)\rightarrow Q(X\_{i},τ,E)$ is $Soft contra J continuous$ then $Q\_{i}f:(X,τ,E)\rightarrow (X\_{i},τ,E)$ is $Soft contra J continuous$ for each $i\in I$, where $Q\_{i}$ is the $Soft projection$ of $Q(X\_{i},τ,E)$ onto $(X\_{i},τ,E)$.

**Proof:** It has been verified by the combination of facts that $Soft projection$ is continuous.

**Theorem 4.19:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is a $Soft surjective J open map$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is a map such that their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft contra J continuous$, then $g$ is $Soft contra J continuous$.

**Proof:** Let $(H,R)$ be a $Soft closed$ set in $(Z,η,R)$. Since $g∘f$ is a $Soft contra J continuous$, $(g∘ f)^{-1}(H,R)=f^{-1}(g^{-1}(H,R))$ is $Soft J open$ in $(X,τ,E)$. Because $f$ is $Soft surjective$ and $Soft J open$, $f((g∘f)^{-1}(H,R))=(g^{-1}(H,R))$ is $Soft J open$ in $(Y,σ,K)$. Thereon $g$ is $Soft contra J continuous$.

**Theorem 4.20:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft contra J continuous$ and $(Y,σ,K)$ is $Soft regular$ then $f$ is $Soft J continuous$.

**Proof:** Let $(x,e)$ be an arbitrary $Soft point$ of $\tilde{X}$ and $(F,K)$ be a $Soft open$ set of $(Y,σ,K)$ containing $f((x,e))$. Since $(Y,σ,K)$ is $Soft regular$, there exists a $Soft open$ set $(V,K)$ in $(Y,σ,K)$ containing $f((x,e))$ such that $Cl(V,K)\tilde{⊂}(F,K)$. Now, $Cl(V,K)$ is a $Soft closed$ set in $(Y,σ,K)$ containing $f((x,e))$ and $f$ is $Soft contra J continuous$. Therefore, by Theorem 4.11 there exists $(U,E)\tilde{\in }SJPO(X,τ,E)$ such that $f(V,K)\tilde{⊂}Cl(V,K)$. Then $f(U,E)\tilde{⊂}(F,K)$. Hence $f$ is $Soft J continuous$.

**Proposition 4.21:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $contra Soft semi continuous$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is $Soft contra continuous$ $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft J continuous$.

**Proof:** Let $(H,R)$ be a any $Soft closed$ set in $(Z,η,R)$. Since $g$ is $Soft contra continuous$, $g^{-1}(H,R)$ is $Soft open$ in $(Y,σ,K)$. Since $f$ is $contra Soft semi continuous$ $f^{-1}(g^{-1}\left(H,R\right))$ is $Soft semi closed$ set in $(X,τ,E)$. Because each one of the $Soft semi closed$ set is $Soft J closed$, $f^{-1}(g^{-1}\left(H,R\right))$ is $Soft J closed$ set in $(X,τ,E)$. Thus $g∘f$ is $Soft J continuous$.

**Theorem 4.22:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be any two maps then

1. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft J irresolute$ if both $f$ and $g$ are $Soft J irresolute$.
2. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft J continuous$ if $f$ is $Soft J irresolute$ and $g$ is $Soft J continuous$.

**Proof:**

1. Let $(V,R)$ be a $Soft J closed$ set in $(Z,η,R)$. Since $g$ is $Soft J irresolute$, $g^{-1}(V,R)$ is $Soft J closed$ in $(Y,σ,K)$. Because $f$ is $Soft J irresolute$ $f^{-1}\left(g^{-1}\left(V,R\right)\right)=\left(g∘f\right)^{-1}(V,R)$ is $Soft J closed$ in $(X,τ,E)$. So, $g∘f$ is $Soft J irresolute$.
2. Let $(V,R)$ be a $Soft closed$ set in $(Z,η,R)$. Since $g$ is $Soft J continuous$ $g^{-1}(V,R)$ is $Soft J closed$ in $(Y,σ,K)$. Since $f$ is $Soft J irresolute$ $f^{-1}\left(g^{-1}\left(V,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft J closed$ in $(X,τ,E)$. So, $g∘f$ is $Soft J continuous$.

**Theorem 4.23:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be any two $Soft maps$ then

1. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft continuous$ if $f$ is $Soft strongly J continuous$ and $g$ are $Soft J continuous$.
2. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft strongly J continuous$ if both $f$ and $g$ are $Soft strongly J continuous$.
3. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft strongly J continuous$ if $f$ is $Soft continuous$ and $g$ is $Soft strongly J continuous$.
4. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft continuous$ if $f$ is $Soft strongly J continuous$ and $g$ is $Soft continuous$.
5. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft J irresolute$ if $f$ is $Soft J continuous$ and $g$ is $Soft strongly J continuous$.

**Proof:**

1. Let $(H,R)$ be a $Soft closed$ set in $(Z,η,R)$. Since $g$ is $Soft J continuous$ $g^{-1}(H,R)$ is $Soft J closed$ in $(Y,σ,K)$. Since $f$ is $Soft strongly J continuous$ $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft closed$ in $(X,τ,E)$. So, $g∘f$ is $Soft continuous$.
2. Let $(H,R)$ be a $Soft J closed$ set in $(Z,η,R)$. Since $g$ is $Soft strongly J continuous$ $g^{-1}(H,R)$ is $Soft closed$ in $(Y,σ,K)$. By Result 2.7,$ g^{-1}(H,R)$ is $Soft J closed$ set in $(Y,σ,K)$. Because $f$ is also $Soft strongly J continuous$, $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft closed$ in $(X,τ,E)$. Thus $g∘f$ is $Soft strongly J continuous$.
3. Let $(H,R)$ be a $Soft J closed$ set in $(Z,η,R)$. Since $g$ is $Soft strongly J continuous$ $g^{-1}(H,R)$ is $Soft closed$ in $(Y,σ,K)$. Since $f$ is $Soft continuous$ $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft closed$ in $(X,τ,E)$. Thus $g∘f$ is $Soft strongly J continuous$.
4. Let $(H,R)$ be a $Soft closed$ set in $(Z,η,R)$. Since $g$ is $Soft continuous$ $g^{-1}(H,R)$ is $Soft closed$ in $(Y,σ,K)$. By Result 2.7, $g^{-1}(H,R)$ is $Soft J closed$ set in $(Y,σ,K)$. As $f$ is $Soft strongly J continuous$, $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft closed$ in $(X,τ,E)$. So, $g∘f$ is $Soft continuous$.
5. Let $(H,R)$ be a $Soft J closed$ set in $(Z,η,R)$. Because $g$ is $Soft strongly J continuous$ $g^{-1}\left(H,R\right)$ is $Soft closed$ in $(Y,σ,K)$. Since $f$ is $Soft J continuous$ $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft J closed$ in $(X,τ,E)$. Thus $g∘f$ is $Soft J irresolute$.

**Theorem 4.24:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be any two $Soft$ maps then

1. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft perfectly continuous$ if $f$ is $Soft strongly continuous$ and $g$ are $Soft perfectly continuous$.
2. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft perfectly J continuous$ if both $f$ and $g$ are $Soft perfectly J continuous$.
3. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft perfectly J continuous$ if $f$ is $Soft perfectly J continuous$ and $g$ is $Soft J irresolute$.

**Proof:**

1. Let $(H,R)$ be a $Soft J closed$ set in $(Z,η,R)$. Since $g$ is $Soft perfectly J continuous$ $g^{-1}\left(H,R\right)$ is $Soft clopen$ in $(Y,σ,K)$. Since $f$ is $Soft strongly J continuous$ $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft clopen$ in $(X,τ,E)$. Thus $g∘f$ is $Soft perfectly continuous$.
2. Let $(H,R)$ be a $Soft J closed$ set in $(Z,η,R)$. Since $g$ is $Soft perfectly J continuous$ $g^{-1}\left(H,R\right)$ is $Soft clopen$ in $(Y,σ,K)$. By Result 2.7, $g^{-1}\left(H,R\right)$ is $Soft J closed$ set in $(Y,σ,K)$. Now, $f$ is also $Soft perfectly J continuous$, then $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft clopen$ in $(X,τ,E)$. Thus$ g∘f$ is $Soft perfectly J continuous$.
3. Let $(H,R)$ be a $Soft J closed$ set in $(Z,η,R)$. Since $g$ is $Soft J irresolute$, $g^{-1}\left(H,R\right)$ is $Soft J closed$ set in $(Y,σ,K)$. Since $f$ is $Soft perfectly J continuous$ $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft clopen$ in $(X,τ,E)$. So, $g∘f$ is $Soft perfectly J continuous$.

**Theorem 4.25:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be any two $Soft$ maps then

1. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft perfectly continuous$ if $f$ is $Soft perfectly J continuous$ and $g$ are $Soft strongly J continuous$.
2. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft strongly J continuous$ if $g$ is $Soft perfectly J continuous$ and $f$ is $Soft continuous$.

**Proof:**

1. Let $(H,R)$ be a $Soft J closed$ set in $(Z,η,R)$. Since $g$ is $Soft strongly J continuous$ $g^{-1}\left(H,R\right)$ is $Soft closed$ set in $(Y,σ,K)$. By Result 2.7, $g^{-1}\left(H,R\right)$ is $Soft JP closed$ set in $(Y,σ,K)$. Since $f$ is $Soft perfectly J continuous$ $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft clopen$ in $(X,τ,E)$. Thus $g∘f$ is $Soft perfectly continuous$.
2. Let $(H,R)$ be a $Soft J closed$ set in $(Z,η,R)$. Since $g$ is $Soft perfectly J continuous$ $g^{-1}\left(H,R\right)$ is $Soft clopen$ in $(Y,σ,K)$. Since $f$ is $Soft continuous$ $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft clopen$ in $(X,τ,E)$.Thus $g∘f$ is $Soft perfectly J continuous$.

**Theorem 4.26:** Let $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be any two maps then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft perfectly J continuous$ if $g$ is $Soft strongly continuous$ and $g$ is $Soft perfectly J continuous$.

**Proof:** Let $(H,R)$ be a $Soft J closed$ set in $(Z,η,R)$. Since $g$ is $Soft strongly continuous$ $g^{-1}\left(H,R\right)$ is $Soft closed$ in $(Y,σ,K)$. Since $f$ is $Soft perfectly J continuous$ $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft J clopen$ in $(X,τ,E)$. Thus $g∘f$ is $Soft perfectly J continuous$.

**Theorem 4.27:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft strongly J continuous$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is $Soft contra J continuous$ then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft contra continuous$.

**Proof:** Let $(H,R)$ be a $Soft closed$ set in $(Z,η,R)$. Since $g$ is $Soft contra J continuous$ $g^{-1}\left(H,R\right)$ is $Soft J open$ in $(Y,σ,K)$. Since $f$ is $Soft strongly J continuous$ $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft open$ in $(X,τ,E)$. Thus $g∘f$ is $Soft contra continuous$.

**Theorem 4.28:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is $Soft perfectly J continuous$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is $Soft contra J continuous$ then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is $Soft contra J continuous$.

**Proof:** Let $(H,R)$ be a $Soft closed$ set in $(Z,η,R)$. Since $g$ is $Soft contra J continuous$ $g^{-1}\left(H,R\right)$ is $Soft J open$ in $(Y,σ,K)$. Since $f$ is $Soft perfectly J continuous$ $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is $Soft clopen$ in $(X,τ,E)$. Then by Result 2.7, $\left(g∘f\right)^{-1}(H,R)$ is $Soft J open$ in $(X,τ,E)$. Thus $g∘f$ is $Soft contra J continuous$.­

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