**Chapter 5**

**Combined Error Minimization Model (CEMM)**

*This paper presents a novel approach for short term wind speed forecasting using combined error minimization technique for optimally selecting combinational models. A mathematical expression is derived for minimizing forecast error based on error output of each combinational model. Intercept and combination coefficient of model at each interval are used as penalty factor for optimizing error and regression coefficient. Optimization algorithm is used to minimize forecast MAPE and maximize regression coefficient.*

**5.1 Introduction**

Combined forecasts created by assigning equal weights to individual forecasts is acceptable but, assigning greater weight to the set of forecasts which seemed to contain lower errors. There are many ways to determine these weights, and goal is to choose a forecast which is likely to yield lesser error for the combinational forecasts. Forecast combinations have frequently been found in empirical studies to produce better forecasts on average than methods based on the ex-ante best individual forecasting model. Moreover, simple combinations that ignore correlations between forecast errors often dominate more refined combination schemes aimed at estimating the theoretically optimal combination weights. In this chapter analysis of theoretically defined factors that determine the advantages from combining forecasts is carried out. Although the reasons for the success of simple combination schemes are poorly understood, we discuss several possibilities related to model misspecification, instability (non-stationarities) and estimation error in situations where the numbers of models is large relative to the available sample size. We discuss the role of combinations under asymmetric loss and consider combinations of point, interval and probability forecasts.

**5.2 Derivation for Combined Error Minimization Model**

Let be wind speed forecast by individual model, where i is model number from 1….m at time t ϵ H provided by m different forecast models, where H is forecast horizon. Total wind speed forecast error is given by

|  |  |  |
| --- | --- | --- |
|  |  | (5.1) |

Where,

|  |  |  |
| --- | --- | --- |
|  | , measured by MSE | |
|  | , measured by ME | |
|  | , measured by SE | |
|  | | (5.2) |
| Where, | is variance of | |

Two types of errors in (1) can be combined to form MSE

|  |  |  |
| --- | --- | --- |
|  | . | (5.3) |

Now assuming that, can be represented by polynomial order with time varying coefficients and is given by

|  |  |  |
| --- | --- | --- |
|  |  | (5.4) |

Where, j is order of polynomial.

The total error in (5.4) can be determined by formulating an improved prediction , the error minimization model (EMM) is given by

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | (5.5) |

Where, is improved forecast in a linear combination process. Equation (5) can be expressed for each model forecast as

|  |  |  |
| --- | --- | --- |
|  |  | (5.6) |

Where, = intercept, = combination coefficient of the model at time t assuming without any restrictions on coefficients.

Substituting (5.5) in (5.6) and rearranging terms, we obtain combinational forecast value of wind speed as given by:

|  |  |  |
| --- | --- | --- |
|  |  | (5.7) |

Combinational forecast value in (5.7) is written as:

|  |  |  |
| --- | --- | --- |
|  |  | (5.8) |

Where,

in (5.8) represents Combinational Error Minimization Model(CEMM). Total error is considered to be contributing for forecasts of individual model. It is to be noted that, combined forecast cannot be determined by linear regression consisting time varying coefficients of CEMM. Hence, have to be estimated separately. It is known [12-14] that, estimation of time varying coefficients is necessary to adapt to changing situations of model. A dynamic Levenberg Marquardt algorithm in [15-16] is used to estimate these time varying coefficients. Moving Window principle is used to update coefficients at each stage. The state equation for time varying state vector is given by:

|  |  |  |
| --- | --- | --- |
|  |  | (5.9) |

Where,

is system dynamic, I is identity matrix, and

is coupled and but not directly observable process given by

|  |  |  |
| --- | --- | --- |
|  |  | (5.10) |

Where, is observation vector at time t.

If , then eq.(4) is expressed by

|  |  |  |
| --- | --- | --- |
|  | **.** | (5.11) |

If and are assumed to follow multivariate normally distributed and and , and as well as and are independently distributed for , then common filter algorithm can be used for estimation of optimal . This is influenced strongly by measurement and model noise which is implemented in this paper by the following covariance matrices of state estimation given by

|  |  |  |
| --- | --- | --- |
|  |  | (5.12) |
|  |  | (5.13) |
|  |  | (5.14) |

Where, is time variant covariance matrix, is measurement noise covariance matrix and is error covariance of the state estimation. Calculation of error is further simplified by including forgetting factor which simplifies estimation of error covariance to:

|  |  |  |
| --- | --- | --- |
|  |  | (5.15) |

Value of is selected near to 1 so as to avoid the estimation of . For the combination coefficient , dynamic regression is considered. Using historical wind speed observations for m models, correction and combination coefficients are determined. Initially EMM is used to improve prediction values of each model for the next interval. Subsequently, are used in different CEMM approaches. The performance of CEMM forecast is estimated using Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC). AIC estimates information loss when the probability distribution f associated with the true (generating) model is approximated by probability distribution g associated with model that is to be evaluated. For a set of orders of any model, AIC and BIC of model are given by [17-18]

|  |  |  |
| --- | --- | --- |
|  |  | (5.16) |
|  |  | (5.17) |

Where , n is size of data and constant term determined by least square estimation procedure.

Number of parameters to be estimated,

Recursively minimize information lost in the model,

|  |  |  |
| --- | --- | --- |
|  | Minimize (= | (5.18) |

Subject to

Final prediction error (FPE) as in [17] is given by:

|  |  |
| --- | --- |
| Minimize | (5.19) |
| Subject to | (5.20) |
| Where,v- Loss Function |  |

**Fig. 5.1: A novel algorithm for combinational models using CEMM**

No

t=t+1, it=it+1

Yes

Yes

Models

f1= ARIMA

f2=TRFU-ARIMA

f3=GARCH

f4=Wavelet

f5= ANN

f6= Fuzzy logic

f7= ANFIS

f8= SVM

Test passed?

Parameters estimation

Models *f1, f2….f8*

Initial Estimation of AIC, BIC, FPE

Error and Co variance matrices at each interval Δws (m,t) eq.(12-15)

Error model *f(*Δws(m,t)) Eq. (2)

Estimate AIC,BIC,FPE for EM eq:(16-20)

Error distribution ACF, PACF, Spectrum

Error minimization model EMM, Eq.(8)

Assigning Penalty factor for *f1, f2….f8* by using EMM.Rule formation for optimum weightage

Optimize combined f(*f1, f2….f8)t* Formulate final equation of optimization

Data, site, horizon

Stat, Data conditioning statistics, data tests

t=h

it=itmax

Store error f((*fopt) )t* optimum

Update error

t-test, h-test, R-test

Use *fopt* for forecast

No

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 5.1 Initial Error Matrix for Individual Model of EMM** | | | | | | | | |  | **Table 5.2 Final Error Matrix for Individual Model of EMM** | | | | | | | | |
|  | | | | | | | | |  | | | | | | | | |
| Hr | M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 | Hr | M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 |
| 1 | 2.73 | 3.41 | 3.81 | 2.32 | 2.20 | 2.12 | 1.71 | 1.37 | 1 | 1.30 | 1.63 | 1.81 | 1.11 | 1.05 | 1.01 | 0.81 | 0.65 |
| 2 | 2.60 | 3.26 | 3.63 | 2.21 | 2.10 | 2.02 | 1.63 | 1.30 | 2 | 1.24 | 1.55 | 1.73 | 1.05 | 1.00 | 0.96 | 0.78 | 0.62 |
| 3 | 2.98 | 3.73 | 4.16 | 2.53 | 2.41 | 2.31 | 1.86 | 1.49 | 3 | 1.42 | 1.78 | 1.98 | 1.21 | 1.15 | 1.10 | 0.89 | 0.71 |
| 4 | 2.57 | 3.22 | 3.59 | 2.19 | 2.08 | 1.99 | 1.61 | 1.29 | 4 | 1.22 | 1.53 | 1.71 | 1.04 | 0.99 | 0.95 | 0.77 | 0.61 |
| 5 | 2.48 | 3.10 | 3.46 | 2.11 | 2.00 | 1.92 | 1.55 | 1.24 | 5 | 1.18 | 1.48 | 1.65 | 1.00 | 0.95 | 0.91 | 0.74 | 0.59 |
| 6 | 2.31 | 2.89 | 3.22 | 1.96 | 1.87 | 1.79 | 1.44 | 1.16 | 6 | 1.10 | 1.38 | 1.53 | 0.94 | 0.89 | 0.85 | 0.69 | 0.55 |
| 7 | 2.60 | 3.26 | 3.63 | 2.21 | 2.10 | 2.02 | 1.63 | 1.30 | 7 | 1.24 | 1.55 | 1.73 | 1.05 | 1.00 | 0.96 | 0.78 | 0.62 |
| 8 | 2.52 | 3.15 | 3.52 | 2.14 | 2.03 | 1.95 | 1.58 | 1.26 | 8 | 1.20 | 1.50 | 1.67 | 1.02 | 0.97 | 0.93 | 0.75 | 0.60 |
| 9 | 2.52 | 3.15 | 3.52 | 2.14 | 2.03 | 1.95 | 1.58 | 1.26 | 9 | 1.20 | 1.50 | 1.67 | 1.02 | 0.97 | 0.93 | 0.75 | 0.60 |
| 10 | 2.98 | 3.73 | 4.16 | 2.53 | 2.41 | 2.31 | 1.86 | 1.49 | 10 | 1.42 | 1.78 | 1.98 | 1.21 | 1.15 | 1.10 | 0.89 | 0.71 |
| 11 | 2.73 | 3.41 | 3.81 | 2.32 | 2.20 | 2.12 | 1.71 | 1.37 | 11 | 1.30 | 1.63 | 1.81 | 1.11 | 1.05 | 1.01 | 0.81 | 0.65 |
| 12 | 2.69 | 3.36 | 3.75 | 2.28 | 2.17 | 2.08 | 1.68 | 1.34 | 12 | 1.28 | 1.60 | 1.79 | 1.09 | 1.03 | 0.99 | 0.80 | 0.64 |
| 13 | 2.60 | 3.26 | 3.63 | 2.21 | 2.10 | 2.02 | 1.63 | 1.30 | 13 | 1.24 | 1.55 | 1.73 | 1.05 | 1.00 | 0.96 | 0.78 | 0.62 |
| 14 | 2.86 | 3.57 | 3.99 | 2.43 | 2.31 | 2.21 | 1.79 | 1.43 | 14 | 1.36 | 1.70 | 1.90 | 1.16 | 1.10 | 1.05 | 0.85 | 0.68 |
| 15 | 2.52 | 3.15 | 3.52 | 2.14 | 2.03 | 1.95 | 1.58 | 1.26 | 15 | 1.20 | 1.50 | 1.67 | 1.02 | 0.97 | 0.93 | 0.75 | 0.60 |
| 16 | 2.60 | 3.26 | 3.63 | 2.21 | 2.10 | 2.02 | 1.63 | 1.30 | 16 | 1.24 | 1.55 | 1.73 | 1.05 | 1.00 | 0.96 | 0.78 | 0.62 |
| 17 | 2.77 | 3.47 | 3.87 | 2.36 | 2.24 | 2.15 | 1.73 | 1.39 | 17 | 1.32 | 1.65 | 1.84 | 1.12 | 1.07 | 1.02 | 0.83 | 0.66 |
| 18 | 2.31 | 2.89 | 3.22 | 1.96 | 1.87 | 1.79 | 1.44 | 1.16 | 18 | 1.10 | 1.38 | 1.53 | 0.94 | 0.89 | 0.85 | 0.69 | 0.55 |
| 19 | 2.90 | 3.62 | 4.04 | 2.46 | 2.34 | 2.25 | 1.81 | 1.45 | 19 | 1.38 | 1.73 | 1.93 | 1.17 | 1.11 | 1.07 | 0.86 | 0.69 |
| 20 | 2.56 | 3.20 | 3.57 | 2.18 | 2.07 | 1.99 | 1.60 | 1.28 | 20 | 1.22 | 1.53 | 1.70 | 1.04 | 0.99 | 0.95 | 0.76 | 0.61 |
| 21 | 2.98 | 3.73 | 4.16 | 2.53 | 2.41 | 2.31 | 1.86 | 1.49 | 21 | 1.42 | 1.78 | 1.98 | 1.21 | 1.15 | 1.10 | 0.89 | 0.71 |
| 22 | 3.23 | 4.04 | 4.51 | 2.75 | 2.61 | 2.51 | 2.02 | 1.62 | 22 | 1.54 | 1.93 | 2.15 | 1.31 | 1.24 | 1.19 | 0.96 | 0.77 |
| 23 | 3.70 | 4.62 | 5.16 | 3.14 | 2.98 | 2.87 | 2.31 | 1.85 | 23 | 1.76 | 2.20 | 2.46 | 1.50 | 1.42 | 1.36 | 1.10 | 0.88 |
| 24 | 2.73 | 3.41 | 3.81 | 2.32 | 2.20 | 2.12 | 1.71 | 1.37 | 24 | 1.3 | 1.63 | 1.81 | 1.11 | 1.05 | 1.01 | 0.81 | 0.65 |
| **Avg.** | **2.73** | **3.41** | **3.81** | **2.32** | **2.20** | **2.12** | **1.71** | **1.36** | **Avg.** | **1.3** | **1.62** | **1.81** | **1.1** | **1.05** | **1.01** | **0.81** | **0.65** |

**Summary of**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 5.3 Summary of Model Parameter to Be Estimated** | | | | | | | |
| **Sl.** | **Model Name** | **Parameters** |  | **Sl.** | **Model Name** | | **Parameters** |
| 1. | ARIMA  (p,d,q) | θ, Ø, p, d, q, AICp-q, BICp-q |  | 5. | Fuzzy Logic | | wi, bj, μ |
| 2. | GARCH | α0, αi, θ, p, q |  | 7. | ANFIS | | wi, bj, μ |
| 3. | TRFU-ARIMA | d, ω, δ, θ, b, r |  | 8. | SVM | (a)RBF: | xi, ai, αi, yi, |
| 4. | Wavelet Transform | a, b, ψ |  | (b)Polynomial | : yi, xi, d, b, αi |
| 5. | Neural Network | wk1…wkm , uk, bk, Ø |  | (c)Two Layer | : wi, N, α, b, v, c |

**Algorithm for Combined Model with Error Minimization Model**

Step 0: Select train, test and validation data, Plot Data for observation. Divide data as train, test and Validation.(50:25:25)%. Set it=itmax=100

Step 1: Find Statistics, Calculate ACF, PACF, and Spectrum of data to find stationarity, outliers and any missing intervals.

Step 2: Estimation of parameters of Models to develop functions of ARIMA, TRFU-ARIMA, GARCH, Wavelet, ANN, Fuzzy logic, ANFIS and SVM models (f1,f2,….,f8). Initial Optimization of Functions using Eq. Annexure I to VIII.

Step 3: Initial Estimation of AIC, BIC and FPE of models Eq.(12) to (15)

Step 4: Error Model for f1,f2,…,f8 . f(Δws(m,t)) Eq.(2).

Step 5: Analysis of Distribution of Error, ACF, PACF and Spectrum of Error for calculating orders of Error Minimization model.

Step 6: Error Minimization Model EMM using Eq.(6). Form Initial Matrices for Error and Covariance of EMM.

Step 7: Assigning Penalty Factor for functions using EMM and Covariance matrices.

Step 8: Rule Formation using Penalty Factor of EMM. Assign Weightage to each model.

Step 9: Optimize each functions f1,f2,…,f8 using EMM matrix and Optimization Functions.

Step 10: Increment for next interval and next iteration.

Step 11: Repeat step 2 to step 10 for all intervals of horizon. Formulate final Matrices of Penalty factor, Error Matrices, AIC, BIC and FPE.

Step 12: Store Optimum and rejected model’s values separately at each interval.

Step 13: Forecast wind speed for test data using Optimum Model at Step 12.

Step 14: Test forecast output (t-test, R2, h-test)

Step 15: If tests fail then go to Step 9. Else Go to Step 16

Step 16: Decide the score weightage for models (Score is decided depending upon the Selection criteria- AIC and BIC for Time Series Models, t-test, h-test and regression tests for the models and MAPE and MSE of the forecasted result).

Step 17: Forecast wind speed for Validation data using highest scored model at each interval.

Step 18: Display MAPE, MSE of Individual, Combined, and optimum models

Step 19: Display the forecasted result for individual, optimum and combined model.

Step 20: Display the comparison result of all the models. (Comparison Table and Comparison plot).

**5.3 Results and Discussion**

Ten year historical wind speed data from 2007-17 is used for study. The data is measured with 50m wind-mast at Basaveshwar Engineering College (Autonomous), Bagalkot, Karnataka, India. The system was installed under R & D grant sanctioned by Technical Education Quality Improvement Program (TEQIP) of World Bank. Wind data is recorded using NRG Symphonie Data logger at 10min interval. Average wind speed data of this site is shown in Fig. 5.2. Recorded average wind speed range from 2.97 to 9.38m/s. Maximum wind speed of 20.65m/s and minimum of 0.40m/s are recorded in this period.

**Fig. 5.2: Monthly average wind speed for BEC (A) Site from 2007-17**

## 5.3.1 Results of EMM

Data is divided into three parts as train, test and validation at the ratio of 50:25:25. Seasonal models are developed using winter, summer and rain season data. 24hr and 168hr prediction horizon is used to test the models. Generated initial error matrix at first iteration is used in CEMM for formation of rules, assigning weights and penalty factor to individual forecasts. MAPE and R2 are used as stopping criteria’s. Optimal forecast output obtained after stopping criteria during each iteration is combined and t-test, h-test, R-test will be conducted. The model which passes these tests is used for further forecast. Table 5.1 and 5.2 present initial and final error Matrices for CEMM. It is observed that Model 8(SVM) performed well in all intervals in initial and final iterations. An average of 1.36 at initial iteration and 0.65 at final iteration for M-8 is found. Similarly for 168hr prediction model M-8 performed well after initial and final iterations. M-8 performed well in all intervals in initial and final iteration. An average of 0.99 at initial iteration and 0.80 at final iteration is found during iteration process. Fig. 5.3 and Fig. 5.4 present wind speed forecasting results of individual models and CEMM after final iterations. Results indicate that forecasting accuracy greatly improved after including CEMM along with individual models. Regression for CEMM in Fig. 5.4 is 0.725 which is higher than other individual models.

**Fig. 5.3: Comparison of Individual Model and CEMM**

Curve with black dots in Fig. 4 presents forecasted values after inclusion of CEMM. It is nearly following actual curve. Further, regression coefficient presented in Fig. 5.5 indicates higher linear relationship between actual and forecasted curves for CEMM compared with other individual forecasts without using CEMM.

**Fig. 5.4: Correlation between actual and forecasted wind speed of individual and CEMM**

Validation of this method is further investigated for increased forecast horizon of 168hrs in Fig. 5.5 and Fig. 5.5.

Error of CEMM

**Fig. 5.5: Comparison of Individual Model and CEMM for 168hr prediction**

**Fig. 5.6: Correlation between Actual and Forecasted wind speed for 168 hr prediction**

# 5.3.2 Results of Combinational Model

Method employed for individual methods is further investigated for combinational models using statistical and AI technique methods. 61 combinational models are obtained from the combinational algorithm using 8 model types. The models are ARIMA, TRFU-ARIMA, GARCH, WT, ANN, FL, ANFIS, and SVM. Fig. 5.7 presents 24hr wind speed forecast of combinational method with and without including CEMM.

**Fig. 5.7: Wind Speed forecast result from Combinational and CEMM**

**Fig. 5.8: Wind Speed forecast result from Combinational model**

|  |
| --- |
|  |

**Fig. 5.9: Correlation between actual and forecasted wind speed for Combinational model and CEM model**

The results for 24hr wind speed forecast presented in Fig.5.7-5.9 indicate that, CEMM forecast curve is very near to the actual wind speed compared to the combinational model without CEMM. Further MAPE for CEMM is 5.655% as compared to 5.324% for combinational aggregate model and 5.772% for 6 paired models without CEMM. Regression plot given in Fig. 8 indicate that coefficient of 0.9825 is obtained with CEMM and 0.9029 for aggregate combinational model and 0.8226 for 6 paired model without CEMM. This indicates superiority of CEMM included with combinational model. Fig. 5.10 and Fig.5.11 represent variation of forecasting MAPE for different combinational alternatives obtained from an algorithm. The results clearly indicate that, as the complexity increases accuracy increases for both 24hr and 168hr wind speed forecast.

**Fig. 5.10: Comparison of MAPE values for 24hrs prediction**

**Fig. 5.11: Comparison of MAPE values for 168hrs prediction**

Further inclusion of CEMM with 6 pair model has highest accuracy of 93.57% for 24hr and 89.76% for 168hr prediction.

**5.3.3 Statistical Test Results of CEMM**

Paired t-test, F-test and regression tests are conducted for actual vs forecasted wind speed for paired models of three seasons. Pearson Coefficient of 0.896 is obtained with inclusion of CEMM against 0.846 without CEMM. F-value of 0.888 is achieved with CEMM against 0.818 without CEMM. In regression test, R2 of 0.869 is obtained with CEMM against 0.803 without CEMM. The test results suggest that including of CEMM with paired model improved the forecast performance.

**5.4 Salient Observation from CEMM**

* A novel Combined Error Minimization Model (CEMM) in optimally selecting best model for short term wind speed forecasting is presented.
* A mathematical expression for EMM is derived using forecast error output of various statistical and AI models. Individual model forecasting is compared with CEMM result.
* It is found that results of forecasting performance improved significantly with the inclusion of CEMM. Results reveal that six pair model and aggregate models performed very well with inclusion of CEMM.
* The forecasted wind speed curve nearly follows actual wind speed for CEMM. The t-test, F-test and regression tests have been conducted to validate the results.
* This new approach of short term wind forecasting is helpful for economic dispatch of wind power.

**Appendix**

**A 5.1: Derivation for Optimization Function for Models**

1. **ARIMA Model:** ARIMA model is written as a function of moving average and autoregressive parameters:

|  |  |  |
| --- | --- | --- |
|  |  | (5.21) |

Where,

is AR parameter with p as order

is MA parameter with q as order

d is difference operator

B is Back Shift operator

is sample for model

is white noise assumed normally distributed and stationary

|  |  |  |
| --- | --- | --- |
|  |  | (5.22) |
|  |  | (5.23) |

Recursively updating values

|  |  |  |
| --- | --- | --- |
|  |  | (5.24) |
|  |  | (5.25) |
|  |  | (5.26) |
|  |  | (5.27) |
|  |  | (5.38) |

|  |  |
| --- | --- |
| **(i)Objective Function : , min AIC(p,d,q), min IC(p,d,q)** |  |
| **Subject to Constraints:**  **Where,** | **(5.29)** |
| **(ii) Objective Function :** | **(5.30)** |
| **Subject to** |  |

1. **TRFU ARIMA Model**

The transfer function model TRFU(r,s,b) is:

|  |  |  |
| --- | --- | --- |
|  |  | (5.31) |

Where,

 is a polynomial in B of order s,

 is a polynomial in B of order r,

nt disturbance process can be represented by an ARIMA model.

For the present work only non-seasonal process of the wind speed and temperature is considered. Therefore only non-seasonal difference is taken for the wind speed and temperature data. The transfer function ARIMA model for the present work is given by

|  |  |  |
| --- | --- | --- |
|  |  | (5.32) |

Where,

yt is wind speed time series as output,

xt is temperature time series as input.

|  |  |
| --- | --- |
| **Minimize** | **(5.33)** |
| **Subject to** | **(5.34)** |

1. **GARCH**

ARCH for modeling random disturbance of time series model. can be written as follow:

|  |  |  |
| --- | --- | --- |
|  |  | (5.35) |

Where is conditional variance of

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | (5.36) |

ARCH model, Bollersler (1986) proposed a useful extended model known as generalized ARCH(GARCH) model. The conditional variance equation of GARCH (p, q) model can be shown as

|  |  |  |
| --- | --- | --- |
|  |  | (5.37) |
|  |  | (5.38) |

where p and q, the order, are non-negative integers. Using the lag operator, a GARCH (p, q) model is in the form

|  |  |  |
| --- | --- | --- |
|  |  | (5.39) |

Where,

To keep the second-order stationarity constraint, GARCH models also demand α (B) +θ (B) < 1. It is not a easy work to specify the order of a GARCH model, but luckily, GARCH (1,1) can describe the volatility clusters effect of large numbers of time series

|  |  |
| --- | --- |
|  | **(5.40)** |
| **Subject to,** | **(5.41)** |

1. **Wavelet Transform**

Lets us consider Ψ (t) and Fourier transform of the same is ** .**If **** fulfills the criteria as Equation2, Then Ψ (t) that equation can be said as mother wavelet, which tells the outline of components of the decayed wave.

|  |  |  |
| --- | --- | --- |
|  |  | (5.42) |

After dilation and conversion of Ψ (t),Ψa,b(t) can be obtained as a.

|  |  |  |
| --- | --- | --- |
|  |  | (5.43) |

Here a represents the scale factor,b represents the conversion factor.

|  |  |
| --- | --- |
| **Maximize the function** | **(5.44)** |
| **Subject to,** | **(5.45)** |

1. **ANN**

**Let us consider**  and as input and targeted wind speed series with relation given as

|  |  |  |
| --- | --- | --- |
|  |  | (5.46)  (5.47) |

Weights are updated by

|  |  |  |
| --- | --- | --- |
|  |  | (5.48) |

n-> no. of epochs, -> learning rate, ->momentum (0<<1), are weights,

|  |  |  |
| --- | --- | --- |
|  |  | (5.49) |

Where and are weights for each neuron, g is a constant determined by learning rate.

|  |  |
| --- | --- |
| **Optimize such that** | **(5.50)** |
| **Subjected to** | **(5.51)** |

1. **FUZZY Logic Model**

Rules :

Then:

Where is lable if fuzzy ruless

Where NR – No. of Fuzzy Rules

Now,

, Where NPI is size of input vector

, where NCI input vector size for consequent part of ,

jth Rule output is given by

|  |  |  |
| --- | --- | --- |
|  |  | (5.52) |

Where,

is weight coefficient

is bias term

Gaussian type membership function:

|  |  |  |
| --- | --- | --- |
|  |  | (5.53) |

Rules for Optimization

|  |  |  |
| --- | --- | --- |
|  |  | (5.54) |

Where final output of Fuzzy Logic using weighted average differentiation is is given by:

|  |  |  |
| --- | --- | --- |
|  |  | **(5.55)** |

1. **ANFIS**

Consider one output adaptive network.

|  |  |  |
| --- | --- | --- |
|  |  | (5.56) |

Where, Vector of input variables is I, Set of parameters is S and ANFIS function implementation is.

The function of is the complex function is linear in some elements of S since these components can be distinguished by LSM. Set parameter S can be decayed into twice sets direct sum, S2 is the linear elements of [1].

We contain

|  |  |  |
| --- | --- | --- |
|  |  | (5.57) |

S2 is linear element. The weights , and are estimated as

|  |  |  |
| --- | --- | --- |
|  |  | (5.58) |
|  |  | (5.59) |
|  |  | (5.60) |
|  |  | (5.61) |
|  |  | (5.62) |
|  |  | (5.63) |
|  |  | (5.64) |
|  |  | (5.65) |
|  |  | (5.66) |
|  |  | (5.67) |
|  |  | (5.68) |

Simplified by

|  |  |  |
| --- | --- | --- |
|  |  | (5.69) |

is Mx1 matrix, M- No. of element of consequent part of parameter set

A is PxM matrix , P- no. of N data training modeled to adaptive network.

Y-output vector

Solution for in Minimizing squared error by Least Square Estimator

|  |  |  |
| --- | --- | --- |
|  |  | (5.70) |
|  |  | (5.71) |

By using Recursive Least Square Estimator

|  |  |  |
| --- | --- | --- |
|  |  | **(5.72)** |
|  | **For** | **(5.73)** |

Where,

is row vector of Matirx A

is ith element of y

is covariance matrix =

1. **SVM**

The support vector machine implements the SRM (Statistical Risk Minimization) principle. Given the pair of values training

|  |  |  |
| --- | --- | --- |
|  |  | (5.74) |

Can be separated by hyper plane

|  |  |  |
| --- | --- | --- |
|  |  | (5.75) |
|  |  | (5.76) |
|  |  | (5.77) |
|  |  | (5.78) |

|  |  |
| --- | --- |
| **Minimize**  **Such that** | **(5.79)**  **(5.80)** |
| **Where h is VC dimension**  **is margin separating hyperplane** | **(5.81)** |
|  | **(5.82)** |
|  | **(5.83)** |

Solution for this by Lagrange Function by solving equation

|  |  |  |
| --- | --- | --- |
|  |  | (5.84) |

Subject to

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | (5.85) |
|  | |  | (5.86) |
|  | |  | (5.87) |
|  | |  | (5.88) |
| **Maximize in nonnegative quadrant** | | **(5.89)** |
| **Under the constraint** | | **(5.90)** |