**SOLUTION OF SECOND KIND LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION VIA MOHAND DECOMPOSITION METHOD**

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**ABSTRACT:** This chapter presents the "Mohand decomposition method," a novel decomposition technique for figuring out the answer to the second kind linear Volterra integro-differential problem. Three numerical problems provide a detailed description and illustration of the procedure. The current approach is quite effective, according to the results, and it provides the answers without requiring laborious calculating effort.

**KEYWORDS:** Integral Transform; Mohand Transform; Inverse Mohand Transform; Convolution Theorem; Decomposition Method

**MATHEMATICS SUBJECT CLASSIFICATION:** 44A05, 44A35, 44A15, 45A05, 45J05.

**INTRODUCTION:** Integral and integro-differential equations are widely used in the development of mathematical models for the solutions of various problems, including those involving electric circuits, mechanical vibration, heat transfer, compartment problems, and bacterial growth [1-2]. In order to find the answers to issues in physics, chemical science, mathematics, mechanics, and medical science, researchers now apply a variety of integral transformations [3-22]. Utilizing Kamal, Mahgoub, Sadik, Aboodh, Mohand, Elzaki, Laplace-Carson, Laplace, Sawi, Sumudu, and Shehu transforms, researchers [23–35] were able to fully solve the first and second kinds of Volterra integro-differential equation problems. Comparative analyses of the Mohand and other transformations were conducted by Aggarwal and other researchers [36–41]. Aggarwal et al. [42-43] defined the Mohand transforms of Bessel’s and error functions in their study. Aggarwal and Gupta [44] developed the duality relations of Mohand and other various integral transforms.

This chapter's primary goal is to use the Mohand decomposition method to find the solution of the second kind linear Volterra integro-differential problem.

**DEFINITION OF MOHAND TRANSFORM**: The Mohand transform of the function for all is defined as [44]:

where is the Mohand transform operator. Standard properties of Mohand transform and Mohand transform of useful mathematical functions are presented in Table: 1 and Table: 2 respectively (See Table: 1 & Table: 2).

**TABLE: 1 USEFUL PROPERTIES OF MOHAND TRANSFORM [42]**

|  |  |  |
| --- | --- | --- |
| S.N. | Name of Property | Mathematical Form |
| 1 | Linearity |  |
| 2 | Change of Scale |  |
| 3 | Shifting |  |
| 4 | First Derivative |  |
| 5 | Second Derivative |  |
| 6 | Third Derivative |  |
| 7 | nth Derivative |  |
| 8 | Convolution |  |

**Table: 2 MOHAND TRANSFORMS OF USEFUL MATHEMATICAL FUNCTIONS [9, 42-43]**

|  |  |  |
| --- | --- | --- |
| S.N. |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |

**INVERSE MOHAND TRANSFORM:** If then is called the inverse Mohand transform of .

Mathematically, it is represented as, where the operator is called the inverse Mohand transform operator. Inverse Mohand transform of useful mathematical functions are presented in Table: 3 (See Table: 3).

**TABLE: 3 INVERSE MOHAND TRANSFORMS OF USEFUL MATHEMATICAL FUNCTIONS [42-43]**

|  |  |  |
| --- | --- | --- |
| **S.N.** |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |

**MOHAND DECOMPOSITION METHOD FOR SECOND KIND LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION**

The second kind linear Volterra integro-differential equation is given by [25]

(1)

where

For determining the particular solution of equation (1), it is necessary to define the initial conditions .

In this work, we will assume that the kernel of equation (1) is a difference kernel that can be expressed as . Putting this in equation (1), it becomes

(2)

Consider the initial condition as

(3)

Applying Mohand transform on both sides of equation (2), we get

(4)

Using equation (3) in equation (4), we have

(5)

Using convolution theorem of Mohand transform in equation (5), we get

(6)

Operating inverse Mohand transform on both sides of equation (6), we get

(7)

The Mohand decomposition method assumes the solution into infinite series as

(8)

Using equation (8) into equation (7), we have

In general, the recursive relation for the required solution is given by

**NUMERICAL PROBLEMS:** In this section, some numerical problems are considered and solved completely using Mohand transform.

**Problem: 1** Consider the following second kind linear Volterra integro-differential equation

(9)

with (10)

**Solution:** Applying Mohand transform on both sides of equation (9), we get

(11)

Using equation (10) in equation (11), we have

(12)

Using convolution theorem of Mohand transform in equation (12), we get

(13)

Operating inverse Mohand transform on both sides of equation (13), we get

(14)

The Mohand decomposition method assumes the solution into infinite series as

(15)

Using equation (15) into equation (14), we have

From above equation, the recursive relation for the required solution is given by

Using above recursive relation, the first few components of are given as

(16)

(17)

Using equation (15), the required solution of equation (9) with equation (10) is given by

that converges to the exact solution .

**Problem: 2** Consider the following second kind linear Volterra integro-differential equation

(18)

with (19)

**Solution:** Applying Mohand transform on both sides of equation (18), we get

(20)

Using equation (19) in equation (20), we have

(21)

Using convolution theorem of Mohand transform in equation (21), we get

(22)

Operating inverse Mohand transform on both sides of equation (22), we get

(23)

The Mohand decomposition method assumes the solution into infinite series as

(24)

Using equation (24) into equation (23), we have

From above equation, the recursive relation for the required solution is given by

Using above recursive relation, the first few components of are given as

(25)

(26)

Using equation (24), the required solution of equation (18) with equation (19) is given by

that converges to the exact solution .

**Problem: 3** Consider the following second kind linear Volterra integro-differential equation

(27)

with (28)

**Solution:** Applying Mohand transform on both sides of equation (27), we get

(29)

Using equation (28) in equation (29), we have

(30)

Using convolution theorem of Mohand transform in equation (30), we get

(31)

Operating inverse Mohand transform on both sides of equation (31), we get

(32)

The Mohand decomposition method assumes the solution into infinite series as

(33)

Using equation (33) into equation (32), we have

From above equation, the recursive relation for the required solution is given by

Using above recursive relation, the first few components of are given as

(34)

(35)

Using equation (33), the required solution of equation (27) with equation (28) is given by

that converges to the exact solution .

**CONCLUSION**

This chapter effectively determines the solution to the second kind linear Volterra integro-differential problem by using the Mohand decomposition method. The solutions to the issues under consideration show that the Mohand decomposition approach can solve a second-kind linear Volterra integro-differential equation quickly and with little computational effort. In the future, the system of simultaneous linear Volterra integro-differential equations can be solved using the current method.

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