On Some typical kind of Continuity in Soft Topological Space

S. Jackson1, N. Jenifa2, J. Sivasankar3

*1Assistant Professor, 2Research Scholar (Reg. No.* 19112232092010*), 3Research Scholar (Reg. No. 22212232091009) PG and Research Department of Mathematics, V. O. Chidambaram College, (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli - 627 012), Thoothukudi-628 008, Tamil Nadu, India.*

*1*[*jmjjack2008@gmail.com*](mailto:jmjjack2008@gmail.com) *2*[*jenifa.mat@voccollege.ac.in*](mailto:jenifa.mat@voccollege.ac.in)*, 3*[*shivu.san.jr@gmail.com*](mailto:shivu.san.jr@gmail.com)

ABSTRACT

In this paper we have made an attempt to make results on some typical kind of continuous functions of Soft J Open and Soft J Closed sets in Soft Topological spaces. This study also describes the characterization of continuity with reference to our Soft J Open sets in Soft Topological Spaces.

Keywords—Soft J Open set, Soft J Continuous Functions, Soft Strongly J Continuous Functions and Soft Perfectly J Continuous Functions.

# INTRODUCTION

*Soft* set hypothesis was proposed by Molodtsov [4] in 1999 to manage vulnerability in a parametric way. A *Soft* set is a defined group of sets, instinctively *Soft* on the grounds that the limit of the set relies upon the boundaries. One idea of a set is the idea of dubiousness. Molodtsov [6] proposed *Soft* set as a totally nonexclusive numerical instrument for displaying vulnerabilities. There is no restricted condition to the depiction of articles. One of the critical benefits of soft topological spaces lies in their capacity to deal with complex frameworks with deficient or problematic information. They can display questionable conditions, rough thinking, and manage fractional data in a more regular and natural way contrasted with customary topological spaces.

# PRELIMINARIES

## In this section, we discuss the basic definitions and results of Soft set theory that can be considered from previous studies. Throughout this work, refers to an initial universe, is the power set of , denote the set of parameters and denote Soft topological spaces where no Soft separation axioms are assumed until it is explicated.

**Definition 2.1:[6]** A *Soft* set on the universe is deﬁned by the set of ordered pairs , where such that for all . Hence is called an approximate function of the *Soft* set . The value of may be arbitrary, some of them may be empty, some may have non empty intersection.

**Definition 2.2: [2]**

1. A *Soft* set over is said to be Null *Soft* Set denoted by *Fϕ* or if for all , .
2. A *Soft* set over is said to be an Absolute *Soft* Set denoted by *FX* or if for all *,*

**Definition 2.3:[6]** Let be a collection of *Soft* sets over with a fixed set of parameters. Then, is called a ***Soft* topology** on if

1. belongs to .
2. The union of any number of *Soft* sets in belongs to .
3. The intersection of any two *Soft* sets in belongs to .

The triplet is called ***Soft* topological Space** over . The members of are called ***Soft* *open*** sets in and complements of them are called ***Soft* *closed*** sets in .

**Definition 2.4:[3]** A *Soft* set of a *Soft* topological space is known as a ***Soft* J *Closed* set** if when and is *Soft* -*open*. stands for the set of all *Soft* J *closed* sets.

**Definition 2.5.[3]** A *Soft* set of a *Soft* topological space is known as a ***Soft* J *Open* set** if its complement is a *Soft* J *closed* set. stands for the set of all *Soft* J *open* sets.

**Definition 2.6.** **[1,5]** A map is said to be

1. ***Soft* *continuous*** if the inverse image of every *Soft* *open* set in is *Soft* *open* in .
2. ***Soft* semi-*continuous*** if the inverse image of every *Soft* *open* set in is *Soft* semi-*open* in .
3. ***Soft* pre-*continuous*** if the inverse image of every *Soft* *open* set in is *Soft* pre-*open* in .
4. ***Soft* -*continuous*** if the inverse image of every *Soft* *open* set in is *Soft* -*open* in .
5. ***Soft* -*continuous*** if the inverse image of every *Soft* *open* set in is *Soft* -*open* in .
6. ***Soft* g *continuous*** if the inverse image of every *Soft* *open* set in is *Soft* g *open* in .
7. ***Soft* sg *continuous*** if the inverse image of every *Soft* *open* set in is *Soft* sg-*open* in .
8. ***Soft* gs *continuous*** if the inverse image of every *Soft* *open* set in is *Soft* gs-*open* in .
9. ***Soft* gp *continuous*** if the inverse image of every *Soft* *open* set in is *Soft* gp-*open* in .
10. ***Soft* gpr *continuous*** if the inverse image of every *Soft* *open* set in is *Soft* pre-*open* in .
11. ***Soft* -*continuous*** if the inverse image of every *Soft* *open* set in is *Soft* g-*open* in .
12. ***Soft* g-*continuous*** if the inverse image of every *Soft* *open* set in is *Soft* g-*open* in .
13. ***Soft* -*continuous*** if the inverse image of every *Soft* *open* set in is *Soft* -*open* in .

***Soft* JP *continuous*** if the inverse image of every *Soft* *open* set in is *Soft* JP *open* in .

**Result 2.7.[6]**

* + - 1. Each one of the Soft semi closed set remains Soft J closed.
      2. Each one of the Soft closed set remains Soft J closed.
      3. Each one of the Soft set remains Soft J Closed.
      4. Each one of the Soft open set remains Soft J open.
      5. Each one of the Soft semi-open set remains Soft J open.
      6. Each one of the Soft α-open set remains Soft J open.
      7. Each one of the Soft J open set remains Soft gs-open.

# SOFT TOTALLY J CONTINUOUS FUNCTIONS

**Deﬁnition 3.1:** A map is known to be **Soft totally J continuous** if the inverse-image of each one of the Soft open set in is both Soft J closed and Soft J open (i.e Soft J clopen) in .

**Theorem 3.2:** Each one of the Soft perfectly J continuous map is Soft totally J continuous.

**Proof:** Let be Soft perfectly J continuous and be a Soft open set in . Thereon is Soft J open in . Since is Soft Perfectly J continuous, is Soft clopen in . By Result 2.7, is Soft totally J continuous.

**Remark 3.3:** It is observed from the subsequent illustration that the reverse implication of the above theorem is incorrect.

**Example 3.4:** Let Define and as and . Consider the Soft topologies where and are described this way: and where and are described this way: and . Precisely the mapping is Soft totally J continuous. The Soft set defined as is a Soft J open set in . But is not Soft clopen in . Hence is not Soft perfectly J continuous.

**Theorem 3.5:** Each one of the Soft totally J continuous map is Soft J continuous.

**Proof:** Let is Soft totally J continuous and be Soft open set in . Since is Soft perfectly J continuous, is Soft clopen in . Then by proposition 2.1.7 and 2.2.2, is Soft J clopen in . Thereupon is Soft totally J continuous.

**Remark 3.6:** It is observed from the subsequent illustration that the reverse implication of the above theorem is incorrect.

**Example 3.7:** Let Define and as and . Consider the Soft topologies where and are described this way: and where is described this way: . Let be a Soft mapping. Precisely is Soft J continuous but not Soft totally J continuous, because is not Soft J clopen in .

**Theorem 3.8:** Each one of the Soft totally J continuous map is Soft JA continuous.

**Proof:** is Soft J continuous. Let be a Soft closed set in . Then is Soft J closed in . Also, is Soft JA closed in . Thus is Soft JA continuous.

# SOFT CONTRA J CONTINUOUS FUNCTIONS

**Deﬁnition 4.1:** A map is known to be **Soft Contra J continuous** if the inverse-image of each one of the Soft open set in is Soft J closed in .

**Example 4.2:** Let Define and as and . Consider the Soft topologies where and are deﬁned as and where is described this way: . Let be a Soft mapping. Precisely is Soft contra J continuous.

**Proposition 4.3:** If is Soft contra semi continuous then it is Soft contra J continuous.

**Proof:** It is verified by Result 2.7, that each one of the Soft semi closed set is Soft J closed in .

**Proposition 4.4:** If is Soft contra continuous then it is Soft contra J continuous .

**Proof:** It is verified by Result 2.7, that each one of the Soft closed set is Soft J closed in .

**Proposition 4.5:** If is Soft contra -continuous then it is Soft contra J continuous .

**Proof:** It is proved by Result 2.7, that each one of the Soft -closed set is Soft J closed in .

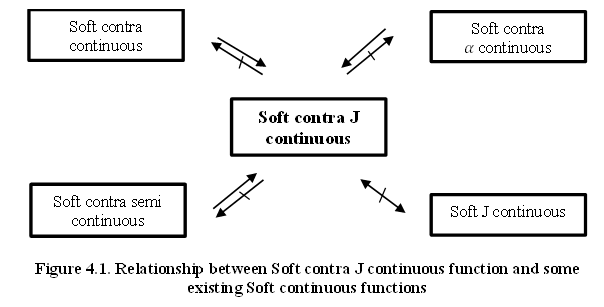
**Result 4.6:** It is observed from the subsequent illustration that the reverse implication of the above propositions 4.3, 4.4, 4.5 are incorrect.

**Example 4.7:** Consider the Soft open set in Example 4.2. Here, is not Soft semi-closed (Soft closed, Soft -closed) in . Hence is Soft contra J continuous but not Soft contra semi continuous (Soft contra continuous, Soft contra -continuous).

**Remark 4.8:** Soft J continuity and Soft contra J continuity are independent. It is observed from the subsequent illustration.

**Example 4.9:**

1. Let Define and as and . Consider the Soft topologies where and are described this way: and where and are described this way: . Precisely the mapping is Soft J continuous. The inverse-image of the Soft open set , is not a Soft J closed set in . Hence is not Soft contra J continuous.
2. Let Define and as and . Consider the Soft topologies where and are deﬁned as and where is described this way: . Let be a Soft mapping. Precisely is Soft contra J continuous. The Soft set defined as is Soft closed in but its inverse-image is not Soft J closed in . Hence is not Soft J continuous.

****

**Lemma 4.10:** The following properties hold for the Soft subsets of a space .

1. and if is Soft open in .

2. then .

**Proof:** The proof is obvious.

**Theorem 4.11:** For a Soft mapping the subsequent properties are equivalent. Assume that is closed under any union and is closed under any intersection.

1. is Soft contra J continuous.

2. The inverse-image of a Soft closed set of is Soft J open.

3. for each one of the Soft subset of of .

4. for each one of the subset of .

**Proof:**

It is evident.

Let be any Soft subset of . Suppose . Then by lemma 4.10, there exists such that . Thus and . Therefore, and . Thereon for every Soft subset of of .

Let be any Soft subset of . Then by (3) and Lemma 4.10, and then .

Let be any Soft open subset of . Therefore, by hypothesis and by Lemma 4.11, . So, . This reveals that is Soft J closed in . Hence is Soft contra J continuous.

**Result 4.12:** The composition of two Soft contra J continuous functions need not be Soft contra J continuous and it is observed from the subsequent illustration.

**Example 4.13:** Let and Define and as and . Consider the Soft topologies where and are described this way: , where is described this way: and where and are described this way: . Let and be two Soft mappings. Precisely and are Soft contra J continuous but their composition is not Soft contra J continuous because is not Soft J closed in .

**Proposition 4.14:** If is Soft J irresolute and is Soft contra J continuous then their composition is Soft contra J continuous.

**Proof:** Let be a Soft closed set in . Since is Soft contra J continuous is Soft J open in . Because is Soft J irresolute, is Soft J open in . So is Soft contra J continuous.

**Proposition 4.15:** If is Soft contra J continuous and is Soft continuous then their composition is Soft contra J continuous.

**Proof:** Let be a Soft closed set in . Since is Soft continuous is Soft closed in . Since is Soft contra J continuous is Soft J open in . Thus is Soft contra J continuous.

**Proposition 4.16:** If is Soft contra semi continuous and is Soft continuous then their composition is Soft contra J continuous.

**Proof:** Let be a Soft closed set in . Since is Soft continuous is Soft closed in . Since is Soft contra semi continuous is Soft semi open in . By Result 2.7, is Soft J open in . Thus is Soft contra J continuous.

**Proposition 4.17:** If is Soft contra -continuous and is Soft continuous then their composition is Soft contra J continuous.

**Proof:** Let be a Soft closed set in . Since is Soft continuous is Soft closed in . Since is Soft contra -continuous, is Soft -open in . By Result 2.7, is Soft J open in . Thus is Soft contra J continuous.

**Theorem 4.18:** Let be any family of Soft topological spaces. If is Soft contra J continuous then is Soft contra J continuous for each , where is the Soft projection of onto .

**Proof:** It has been verified by the combination of facts that Soft projection is continuous.

**Theorem 4.19:** If is a Soft surjective J open map and is a map such that their composition is Soft contra J continuous, then is Soft contra J continuous.

**Proof:** Let be a Soft closed set in . Since is a Soft contra J continuous, is Soft J open in . Because is Soft surjective and Soft J open, is Soft J open in . Thereon is Soft contra J continuous.

**Theorem 4.20:** If is Soft contra J continuous and is Soft regular then is Soft J continuous.

**Proof:** Let be an arbitrary Soft point of and be a Soft open set of containing . Since is Soft regular, there exists a Soft open set in containing such that . Now, is a Soft closed set in containing and is Soft contra J continuous. Therefore, by theorem 4.11 there exists such that . Then . Hence is Soft J continuous.

**Proposition 4.21:** If is contra Soft semi continuous and is Soft contra continuous is Soft J continuous.

**Proof:** Let be a any Soft closed set in . Since is Soft contra continuous, is Soft open in . Since is contra Soft semi continuous is Soft semi closed set in . Because each one of the Soft semi closed set is Soft J closed, is Soft J closed set in . Thus is Soft J continuous.

**Theorem 4.22:** If and be any two maps then

1. is Soft J irresolute if both and are Soft J irresolute.
2. is Soft J continuous if is Soft J irresolute and is Soft J continuous.

**Proof:**

1. Let be a Soft J closed set in . Since is Soft J irresolute, is Soft J closed in . Because is Soft J irresolute is Soft J closed in . So, is Soft J irresolute.
2. Let be a Soft closed set in . Since is Soft J continuous is Soft J closed in . Since is Soft J irresolute is Soft J closed in . So, is Soft J continuous.

**Theorem 4.23:** If and be any two Soft maps then

1. is Soft continuous if is Soft strongly J continuous and are Soft J continuous.
2. is Soft strongly J continuous if both and are Soft strongly J continuous.
3. is Soft strongly J continuous if is Soft continuous and is Soft strongly J continuous.
4. is Soft continuous if is Soft strongly J continuous and is Soft continuous.
5. is Soft J irresolute if is Soft J continuous and is Soft strongly J continuous.

**Proof:**

1. Let be a Soft closed set in . Since is Soft J continuous is Soft J closed in . Since is Soft strongly J continuous is Soft closed in . So, is Soft continuous.
2. Let be a Soft J closed set in . Since is Soft strongly J continuous is Soft closed in . By Result 2.7, is Soft J closed set in . Because is also Soft strongly J continuous, is Soft closed in . Thus is Soft strongly J continuous.
3. Let be a Soft J closed set in . Since is Soft strongly J continuous is Soft closed in . Since is Soft continuous is Soft closed in . Thus is Soft strongly J continuous.
4. Let be a Soft closed set in . Since is Soft continuous is Soft closed in . By Result 2.7, is Soft J closed set in . As is Soft strongly J continuous, is Soft closed in . So, is Soft continuous.
5. Let be a Soft J closed set in . Because is Soft strongly J continuous is Soft closed in . Since is Soft J continuous is Soft J closed in . Thus is Soft J irresolute.

**Theorem 4.24:** If and be any two Soft maps then

1. is Soft perfectly continuous if is Soft strongly continuous and are Soft perfectly continuous.
2. is Soft perfectly J continuous if both and are Soft perfectly J continuous.
3. is Soft perfectly J continuous if is Soft perfectly J continuous and is Soft J irresolute.

**Proof:**

1. Let be a Soft J closed set in . Since is Soft perfectly J continuous is Soft clopen in . Since is Soft strongly J continuous is Soft clopen in . Thus is Soft perfectly continuous.
2. Let be a Soft J closed set in . Since is Soft perfectly J continuous is Soft clopen in . By Result 2.7, is Soft J closed set in . Now, is also Soft perfectly J continuous, then is Soft clopen in . Thus is Soft perfectly J continuous.
3. Let be a Soft J closed set in . Since is Soft J irresolute, is Soft J closed set in . Since is Soft perfectly J continuous is Soft clopen in . So, is Soft perfectly J continuous.

**Theorem 4.25:** If and be any two Soft maps then

1. is Soft perfectly continuous if is Soft perfectly J continuous and are Soft strongly J continuous.
2. is Soft strongly J continuous if is Soft perfectly J continuous and is Soft continuous.

**Proof:**

1. Let be a Soft J closed set in . Since is Soft strongly J continuous is Soft closed set in . By Result 2.7, is Soft JP closed set in . Since is Soft perfectly J continuous is Soft clopen in . Thus is Soft perfectly continuous.
2. Let be a Soft J closed set in . Since is Soft perfectly J continuous is Soft clopen in . Since is Soft continuous is Soft clopen in .Thus is Soft perfectly J continuous.

**Theorem 4.26:** Let and be any two maps then their composition is Soft perfectly J continuous if is Soft strongly continuous and is Soft perfectly J continuous.

**Proof:** Let be a Soft J closed set in . Since is Soft strongly continuous is Soft closed in . Since is Soft perfectly J continuous is Soft J clopen in . Thus is Soft perfectly J continuous.

**Theorem 4.27:** If is Soft strongly J continuous and is Soft contra J continuous then their composition is Soft contra continuous.

**Proof:** Let be a Soft closed set in . Since is Soft contra J continuous is Soft J open in . Since is Soft strongly J continuous is Soft open in . Thus is Soft contra continuous.

**Theorem 4.28:** If is Soft perfectly J continuous and is Soft contra J continuous then their composition is Soft contra J continuous.

**Proof:** Let be a Soft closed set in . Since is Soft contra J continuous is Soft J open in . Since is Soft perfectly J continuous is Soft clopen in . Then by Result 2.7, is Soft J open in . Thus is Soft contra J continuous.­

##### REFERENCES

[1] Arockia Rani, I & Selvi, A 2015, ‘On *Soft* Contra *Continuous* Functions in *Soft* Topological Spaces’, International Journal of Mathematical Trends & Technology, vol. 19, no. 1, pp. 80-90.

[2] Jackson, S & Chitra, S 2021, ‘The New Class of *Closed* and *Open* Sets in *Soft* Topological Spaces’, Journal of the Maharaja Sayajirao University of Baroda, vol. 55, pp. 78-85.

[3] Jackson S and Jenifa N, ‘A New Class of *Soft* Generalized *Closed* Sets in *Soft* Topological Spaces’, Studies in Indian Place Names, vol. 40(70),pp.3380-3385, 2020.

[4] Jun, YB, Kim, HS & Park, CH 2011, ‘Positive Implicative Ideals of BCK-algebras Based on A *Soft* Set Theory’, Bull. Malays. Math. Sci. Soc., vol. 34, no. 2, pp. 345-354.

[5] Maji, PK, Roy, AR & Biswas 2002, ‘An Application of *Soft* Sets in A Decision Making Problem’, Computers and Mathematics with applications, vol. 44, no. 8/9, pp. 1077-1083.

[6] Molodtsov, D 1999, ‘Soft Set Theory First Results’, Computers and Mathematics with Applications, vol. 37, pp. 19-31.

[7] Nakaoka, F & Oda, N 2003, ‘Some Properties of Maximal *Open* Sets,’ International Journal of Mathematics and Mathematical Sciences, vol. 21, pp. 1331-1340.