**THERMAL STRESS DISTRIBUTION IN A TUBE MADE OF PVC/ POLYSTYRENE MATERIAL AND SUBJECTED TO INTERNAL PRESSURE AND MECHANICAL LOAD**

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**ABSTRACT**

This article deals with the study of thermal stress distribution in a tube made of polyvinyl chloride/ polystyrene material and subjected to internal pressure and mechanical load. The transition theory and generalized strain measure are used for finding the governing equation. Mathematical modelling is based on stress –strain relation and equilibrium equation. Analytical solutions are presented in a thick walled tube made of polyvinyl chloride and polystyrene materials. The effects of different pertinent parameters (*i.e.* temperature gradient, load and pressure) are considered for tube made polyvinyl chloride/ polystyrene material, The behaviour of stress distribution, pressure and thermal are investigated. From the obtained results, it is noticed that value of pressure increases with increasing temperature  =0.0175 and decreases with increasing mechanical loads (i.e.  *L*0= 0.1 and 0.2) at the internal surface of a tube made of polyvinyl chloride material and also in a polystyrene material for the initial as well as fully –plastic stage . The Theoretical results are validated by comparing them with those obtained by Gupta et al. [13] after performing some significant calculation examples.

**Keywords –** tube ; load ; stresses ; pressure ; temperature

1. **INTRODUCTION**

Elasto-plastic analyses in thick-walled tubes have attracted a lot of interest due to their important applications in engineering, petrochemical industry, agricultural irrigation, chemical industry, urban construction, and electric power industry. For structural use in bridges, piling pipe, piers, roads, building structures, etc. and also body transport in gas, steam, liquefied petroleum gas, etc. The analytical solutions of stress distribution are given for idealized elasto-plastic by Timoshenko [2] and work hardening by Chadwick [3] for homogeneous materials. Bland [1] has analyzed the problem of thick- walled tubes subjected to uniform pressure and thermal gradient. Gamer et al. [4] achieved the analytical solution of stress distribution in rotating tube by using Tresca’s yield condition. Bree [5] has discussed plastic stress deformation in a closed tube due to the interaction and thermal stresses. Mufit et al. [6] investigated thermal stress distribution in a heat generating tube with yield stress by using Tresca's yield condition and its associated flow rule. Xin *et al.* [7] studied elastic-plastic stress distribution in a functionally graded thick-walled tube subjected to internal pressure by using the assumption of a uniform strain field within the representative volume element and the Tresca yield criterion. Matvienko et al. [10] investigated elastoplastic deformation of dispersion-hardened aluminum tube under external and internal/ external pressure. After that, Matvienko et al. [11] examined mathematical modeling in a tube from dispersion-hardened aluminum alloy with inhomogeneous temperature field..[Qian](https://www.ncbi.nlm.nih.gov/pubmed/?term=Qian%20C%5BAuthor%5D&cauthor=true&cauthor_uid=31947636) et al. [9] developed mechanical properties of highly efficient heat exchange tubes. Gupta et al. [8] has invested the elasto-plastic stress distribution in a cylindrical tube made of steel/copper material and subjected to internal pressure and mechanical load by using transition theory and generalized strain measure. It has been observed , the value of pressure decreases with increasing mechanical loads. Now by applying mechanical load condition, hoop/ radial stress are increasing at the external surface of the contraction/extension region of tube. The objective of this article is to be investigating stress distribution in a thermo mechanical loaded tube made of PVC/ polystyrene material and subjected to uniform pressure. In this present study, we discussed the effect of stress distribution, pressure and Temperature gradient in the contraction and the extension region of a thick walled tube with mechanical loads by using transition theory and generalized strain measure.

1. **MATERIALS USED**

**A. Polyvinyl Chloride (PVC):** Polyvinyl Chloride (PVC or Vinyl) is an [economical and versatile thermoplastic polymer](https://omnexus.specialchem.com/selectors/c-thermoplastics-pvc-polyvinylchloride?src=sg-overview-cnx" \t "_blank) widely used in the building and construction industry to produce door and window profiles, wire and cable insulation, pipes (drinking and wastewater), medical devices, etc. It is a good electrical insulation & vapor barrier properties, good dimensional stability at room temperature and low cost, [flexible](https://omnexus.specialchem.com/polymer-properties/properties/flexibility?src=sg-overview-cnx" \t "_blank) and high impact strength etc.

**B. Polystyrene:** Polystyrene is a versatile plastic used to make a wide variety of consumer products. As a hard, solid plastic, it is often used in products that require clarity i.e. laboratory ware. When combined with various colorants, additives or other plastics, polystyrene is used to make appliances, automobile parts, toys, electronics, gardening pots and equipment etc.

1. **MATEMATICAL MODEL**
2. **Abbreviations and Acronyms**

 - Lame’s constants

 - First strain invariant

*C* - Compressibility factor

 -Internal and external radii

*u*,*v*,*w* -displacement components

 -Poisson’s ratio

 -stress and strain components

*Y ,*  *-*Yield stress

 - Kronecker’s delta

*p*int. - internal surface pressure

*P*i - Pressure required for initial yielding stage

*P*f - Pressure required for fully-plastic stage

 -Load at the external surface

 - Function of *r*

 - Function of *x* and *y*

 - Function of 

Θ - Temperature

1. **Non-dimensional quantities:**

(Radii ratio),  - Radial stress component, - Circumferential stress component , -Mechanical load,  *P* = *p*/*Y* – Pressure, ****-Temperature

**C. Equations**

We consider a thick walled cylindrical tube made of isotropic materials (*i.e.* Polystyrene / Poly vinyl chloride), with an internal radius *a* and external radius *b* (*a*<*b*), and subjected to uniform pressure *p* respectively. Let a uniform temperature be applied at the inner surface of the tube. Further, if we assume that there are no body forces, body couples and couple stresses on the tube, and if only a steady deformation problem is considered as shown in Fig.1.



**Figure. 1** Geometrical configuration of tube made of Polystyrene/ Poly vinyl chloride materials

Basic Governing Equation: The components of displacement in cylindrical polar coordinatesare given [12-16,8 ]:

 (1)

where  is a function of *r*. The Almansi strain components are given [16, 17]:

 ,  ,

 ,  (2)

The stress –strain relations for isotropic material are given [12,14]:

,  (3)

where is the first strain invariant. Substituting Eq. (2) into Eq.(3), we get

 

 (4)

The stress equation of equilibrium is given as:

  (5)

The temperature field satisfying equationis given by the Laplace equation:andat, at  where  is a constant. Solving thisequation,we get:

  (6)

where

Boundary condition*:* The boundary conditions of tube for the contraction /extension region are taken as:

at *r* = *a* andat *r* = *b* (7)

where *l*0 is mechanicalload applied at the external surface of tube made of isotropic materials.

Asymptotic solution at the transition points: Substituting Eq. (4) into Eq. (5) and after integration, we got the following nonlinear integro-differential equation:

 (8)

where *A*0 is the constant of integration*.* Now,differentiating Eq. (8) with respect to *r*, we get:

**  (9)

where (is the function of  and  is the function of *r*) and be the compressibility factor and *v* be the Poisson’s ratio. Transition point *T* from Eq. (9) are and.The transition point  corresponds to extension region and corresponds to contractions region. The resultant force normal to the plane *z* = constant vanishes *i.e.*  The deformation of the tube walls is determined by the magnitude of the applied pressure. If mechanical load is increased, stresses in the wall of the tube increase.

Solution of the problem

(a) Contraction in the tube:The transition point , correspond to contraction in the tube [ 8 16], we define the transition function as:

  (10)

Taking the logarithmic differentiation of Eq. (10) with respect to *r* and using Eq. (9), after that by taking the asymptotic solution  and integrating, we get:

  (11)

where  is a constant of integration. Comparing, Eq. (10) and Eq. (11), we get

 (12)

The yielding stress in tension is given [8-17]:. Now substituting the value of yielding stress condition in Eq. (12), we get

  (13)

Using second boundary condition in Eq. (7) into Eq. (13), we get Further, Eq. (13) become:

  (14)

where  and . Now using first boundary condition in Eq. (7) into Eq. (14), we get



 (15)

Substituting Eq. (15) into Eq. (14) and using Eq. (7), we get stress on the contraction region:

 (16)

 (17)

From Eq. (16) and Eq. (17), we get:

 (18)

Initial yielding stage: From Eq. (18), it has seen that is maximum at the inner surface (*i.e*. *r* = *a*), therefore yielding will take place at the outer surface of the tube and Eq. (18) becomes: (say); where *Y* is the yielding stress on the contraction region. The pressure required for the initial yielding is given by:

 (19)

Eqs. (16), (17) and (19), in non-dimensional form becomes:

 

 (20)

# **where ,,, ,  ,  , and**

**Fully-plastic stage:** Eq. **(20) for fully-plastic stage when  and  becomes:**

,,

 (21)

where  be the pressure required for fully-plastic stage.

(b)Extension in the tube: The transition point , correspond to contraction in the tube [18, 16] we define the transition function as:

 (22)

Taking the logarithmic differentiation of Eq. (22) with respect to *r* and using Eq. (7), after that by taking the asymptotic solution  and integrating, we get:

 (23)

where *B* is a constant of integration. Comparing, Eq. (22) and Eq. (23), we get

 (24)

The yielding stress in tension is given [8-17]:. Now substituting the value of yielding stress condition in Eq. (24), we get

  (25)

Using second boundary condition in Eq. (7) into Eq. (25), we get Further, Eq. (25) become:

  (26)

Now using first boundary condition in Eq. (7) into Eq. (26), we get



 (27)

Substituting Eq. (27) into Eq. (26) and using Eq. (5), we get stress on the contraction region:

 (28)



 (29)

From Eq. (28) and Eq. (29), we get:

 (30)

Initial yielding stage: From Eq. (30), it has seen that is maximum at the outer surface (*i.e*. *r* = *b*), therefore yielding will take place at the outer surface of the tube in the extension region and Eq. (30) becomes:

(say); where  is the yielding stress in the extension region. The pressure required for the initial yielding is given by:

 (31),

Eq. (28)-(29) and (31), in non-dimensional form becomes:

 



and 

 (32)

Fully-plastic stage: Eq. (32), for the fully-plastic stage when, becomes:

,

,

  (33)

where  be the pressure required for fully-plastic stage.

**IV VALIDATION OF RESULTS**

Initial Yielding stage*:* By taking Θ1 →0 into Eq. (20) and Eq. (32), we get:

  (34)

in the contraction region.





 (35)

in the extension region.

Fully-plastic stage*:* By taking Θ1 →0 into Eq. (21) and Eq. (33), we get:

 

,

 (36)

in the contraction/extension region. The present results obtained from Eq. (34) - Eq. (36) are same as given by Gupta et al. [8] in the contraction/extension region. Therefore, the present results are correct and authenticate the validity of the derived solutions. In the present study, we discuss the new addition in thermal condition at the inner surface of the tube.

**A.Figures**

To see the combined effect of stress distribution and pressure in a cylindrical tube made of Poly(vinyl chloride), PVC (say *C* = 0.3333 or *v* = 0.4) and Polystyrene, PS (say *C* = 0.46154 or *v* = 0.35) [17], for the initial/fully plastic stage based upon the following numerical values has been taken: *L*0 = 0, 0.1, 0.2; Θ1  = 0, 0.0175, 0.07; *a* = 1 and *b* = 2 respectively.

**(a)**

**(b)**

**Figure. 2** Graphical comparisons between dimensionless pressure required for yielding/fully-plastic stage versus radii ratio *R*0 =a/b in the (a) contraction (b) extension region

In Figure.2, curves have been drawn between dimensionless pressure required for initial yielding/fully-plastic stage versus radii ratio *R*0 =a/b for the contraction/ extension region and having mechanical loads ( *i.e.* *L*0= 0, 0.1, 0.2) and temperature = 0, 0.0175 respectively. It has been observed that tube made of polyvinyl chloride material requires higher pressure to yield at the internal surface as compared to the tube made of polystyrene material for the initial yielding stage.

Further, the value of pressure increases with increasing temperature  =0.0175 and decreases with increasing mechanical loads (*say L*0= 0.1 and 0.2) at the internal surface of a tube made of polyvinyl chloride material and also in a polystyrene material for the initial as well as fully-plastic stage. Moreover, the thermo mechanical loaded tube made of polyvinyl chloride / polystyrene material requires higher pressure in the contraction region as compared to the extension region at the initial yielding stage.

**(a)**

**(b)**

**Figure. 3.** Dimensionless stress distribution versus radii ratio *R* = *r*/*b* for initial yielding/fully-plastic stage in the with (a)contraction (b) extension region

Figure. 3 are portrayed in order to demonstrate the behaviour of dimensionless stress distribution versus radii ratio *R* = *r*/*b* in the contraction/extension region and having mechanical loads ( *i.e.* *L*0= 0, 0.1, 0.2) and temperature = 0, 0.0175 respectively. It is observed that tube made of polyvinyl chloride material requires maximum hoop stress at the external surface in comparison to tube made of polystyrene material. Further, the values of the hoop / radial stress also increase with increasing temperature  = 0.175 as well as the mechanical load *L*0= 0.1, 0.2 in the contraction/extension region of the tube. Moreover, the extension region in the tube requires maximum hoop stress at the external surface of the initial yielding stage as compared to contraction region and reverse results are obtained with the inclusion of thermal conditions.

**Figure. 4.** Graphical comparisons between dimensionless pressure versus mechanical load at R0 = 0.5 for initial yielding/ fully-plastic stage along the contraction/extension region

Figure. 4 is prepared to illustrate the behaviour of dimensionless pressure versus mechanical load and having temperature  = 0, 0.0175, for the initial yielding /fully-plastic stage at *R*0 = 0.5. It is shown that the contraction/extension region, the value of pressure, decrease with increasing mechanical load *(i.e. L*0 = 0.1, 0.2) and increase with increasing temperature  = 0.175, for initial yielding as well as fully-plastic stage. With increasing mechanical loads, the tube made polyvinyl chloride material requires maximum pressure in comparison to tube made of of polystyrene material.

Figure. 5 demonstrates the behaviour dimensionless pressure versus temperature with mechanical load *L*0 = 0, 0.1, 0.2 for the initial yielding /fully-plastic stage at *R*0 = 0.5. In the contraction region, the value of pressure increases with increasing temperature (*i.e.*  = 0, 0.0175, 0.07) for the initial/fully-plastic stage, but in the case of extension region, the value of pressure neither increases nor decreasing for the initial yielding stage. Furthermore, the values pressure decreases with the mechanical load *L*0 = 0.1, 0.2 in contraction as well as extension region of the tube made of polyvinyl chloride/ polystyrene material .

**Figure. 5.** Graphical comparisons between dimensionless pressure versus temperature at R0 = 0.5 for initial yielding/ fully-plastic stage along the contraction/extension region

**CONCLUSIONS**

In this work, thermal stress distribution and pressure in a tube made of PVC/ polystyrenematerial under internal pressure and mechanical load by using the transition theory is investigated. The problem is designed under uniform pressure, thermal effect and mechanical loads. Solutions of the stress distribution are obtained, for initial and fully-plastic stages, whereas the expressions for non-dimensional pressure versus loads and temperature are evaluated graphically. The main findings can be concluded as follows:

* The polyvinyl chloride material tube requires higher dimensionless pressure to yield at the internal surface in comparison to tube made polystyrene material for the initial yielding stage.
* The value of pressure increases with increasing temperature  =0.0175 and decreases with increasing mechanical loads (*say L*0= 0.1 and 0.2) at the internal surface of a tube made of polyvinyl chloride material and also in a polystyrene material for the initial as well as fully-plastic stage.
* With increasing mechanical loads, the tube made polyvinyl chloride material requires maximum pressure in comparison to tube made of of polystyrene material.
* The values pressure decreases with the mechanical load L0 = 0.1, 0.2 in contraction as well as extension region of the tube made of polyvinyl chloride/ polystyrene material .
* The tube made of polyvinyl chloride material requires maximum hoop stress at the external surface as compared to the tube made of polystyrene material.
* The hoop / radial stress also increase with increasing temperature  = 0.175 as well as the mechanical load *L*0= 0.1, 0.2 in the contraction/extension region of the tube.
* The extension region in the tube requires maximum hoop stress at the external surface of the initial yielding stage as compared to contraction region and reverse results are obtained with the inclusion of thermal conditions.
* The results for Gupta et al. [8] can be obtained by taking →0 in the resulting equations.

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