## BS-ALGEBRAS AND ITS FUZZY IDEAL

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**Abstract:** In this research article, a generalization of B-Algebras called BS-Algebras is introduced. We include fuzzy ideal in BS-Algebras. Some new characterizations were given.

**Keywords:** BS-Algebras, Fuzzy ideal, Homomorphism, Cartesian Product.

# **1. Introduction**

Fuzzy subsets was introduced by L.A.Zadeh[4], many researches investigated the generalization of the notion of fuzzy subset.In 1966, Imai and Iseki established two classes of abstract algebras viz. BCK-algebras and BCI-algebras[2]. J.Neggers and H.S. Kim brought the notion of B-algebras[3] which is a generalisation of BCK-algebras. As an extension, the author newly introduce the notion of BS-algebras, as a generalisation of B- algebras. In this article, the author study the concepts of BS-Algebras with its examples. The author apply the concept of fuzzy ideal in BS-Algebras and find some of their basic properties and explore some algebraic nature of fuzzy ideal in BS-algebras. The homomorphic behaviour of fuzzy ideal of BS-algebras have been investigated. Finally, Cartesian product is also applied in fuzzy ideal of BS-algebras.

# **2. Preliminaries**

**Definition 2.1**. *[3]* A non empty set B-algebra *X* with a constant 0 and a binary operation ∗ satisfying the following conditions

(i) x ∗ x = 0

 (ii) *x* ∗ 0 = *x*

 (iii) (*x* ∗ *y*) ∗ *z* = *x* ∗ (*z* ∗ (0 ∗ *y*)) for all *x, y, z* ∈ *X*

**Definition 2.2.** Let *µ*1 and *µ*2 be any two fuzzy sets of *X*. Then its Intersection is defined by *µ*1 ∩ *µ*2 = *min*{*µ*1(*x*)*, µ*2(*x*)} for all *x* ∈ *X*

**Definition 2.3**. Let *A* = {*µ*1(*x*)*, x* ∈ *X*} and *B* = {*µ*2(*x*)*, x* ∈ *X*} be any two fuzzy sets on *X*. Then The cartesian product *A* × *B* = {*µ*1 × *µ*2(*x, y*) : *x, y* ∈ *X*} which is defined by (*µ*1 × *µ*2)(*x, y*) = *min*{*µ*1(*x*)*, µ*2(*y*)} where *µ*1 × *µ*2 : *X* × *X* → [0*,* 1] for all *x, y* ∈ *X*.

**Definition 2.4**. *[1]* A fuzzy set *µ* of a BS-algebra *X* is called the doubt fuzzy bi-ideal of *X* if

i) *µ*(1) ≤ *µ*(*x*)

ii) *µ*(*y* ∗ *z*) ≤ *max*{*µ*(*x*)*, µ*(*x* ∗ (*y* ∗ *z*))} for all *x, y, z* ∈ *X*

# **3. BS-Algebras**

**Definition 3.1**. A non empty set *X* with a constant 1 and a binary operation ∗ is called a BS-Algebras which satisfies the following conditions

(i) x ∗ x = 1

 (ii) x ∗ 1 = x

 (iii) 1 ∗ *x* = *x*

 (iv) (*x* ∗ *y*) ∗ *z* = *x* ∗ (*z* ∗ (1 ∗ *y*)) for all *x, y, z* ∈ *X*

A binary relation ≤ on BS Algebras *X* can be defined by *x* ≤ *y* iff *x* ∗ *y* = 1

**Example 3.2**. i) A set *X* = {1*, a, b, c*} which satisfies the following table

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| \* | 1 | a | b | c |
| 1 | 1 | a | b | c |
| a | a | 1 | c | b |
| b | b | c | 1 | a |
| c | c | b | a | 1 |

Then (*X,* ∗*,* 1) is a BS-algebra.

1. A set *X* = {1*, a, b*} which satisfies the following table

|  |  |  |  |
| --- | --- | --- | --- |
| \* | 1 | a | b |
| 1 | 1 | a | b |
| a | a | 1 | a |
| b | b | a | 1 |

Then (*X,* ∗*,* 1) is a BS-algebra.

(iii) A set *X* = {1*, a, b, c, d, e*} which satisfies the following table

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| \* | 1 | a | b | c | d | e |
| 1 | 1 | a | b | c | d | e |
| a | a | 1 | b | d | e | c |
| b | b | a | 1 | e | c | d |
| c | c | d | e | 1 | b | a |
| d | d | e | c | a | 1 | b |
| e | e | c | d | b | a | 1 |

If we put *y* = *x* in (*x* ∗ *y*) ∗ *z* = *x* ∗ (*z* ∗ (1 ∗ *y*)), then we have (*x* ∗ *x*) ∗ *z* = *x* ∗ (*z* ∗ (1 ∗ *x*)) → (I)

 ⇒ 1 ∗ *z* = *x* ∗ (*z* ∗ (1 ∗ *x*))

If we put *z* = *x* in (I) then we get 1 ∗ *x* = *x* ∗ (*x* ∗ (1 ∗ *x*)) → (*II*) Using (i) and (I) and *z* = 1 it follows that

1 = *x* ∗ (1 ∗ (1 ∗ *x*))*,* → (*III*)

 we see that the four conditions (i),(ii),(iii) and (iv) are independent.

* 1. A set *X* = {1*, a, b*} which satisfies the following table

|  |  |  |  |
| --- | --- | --- | --- |
| \* | 1 | a | b |
| 1 | 1 | a | 1 |
| a | a | 1 | a |
| b | 1 | a | 1 |

Then the conditions (i) and (iv) hold, but (ii) and (iii) does not hold

since *b* ∗ 1 = 1 ≠ *b*

* 1. The set *X* = {0*,* 1*,* 2} which satisfies the following table

|  |  |  |  |
| --- | --- | --- | --- |
| \* | 1 | a | b |
| 1 | 1 | a | b |
| a | a | a | a |
| b | b | a | b |

The axioms (ii), (iii) and (iv) satisfies

but it does not hold (i) since *a* ∗ *a* = *a* 1 and *b* ∗ *b* = *b* ≠ 1

* 1. Let *X* = {1*, a, b, c*} be a set which satisfies the following table

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| \* | 1 | a | b | c |
| 1 | 1 | a | b | c |
| a | a | 1 | 1 | 1 |
| b | b | 1 | 1 | a |
| c | c | 1 | 1 | 1 |

Then (*X,* ∗*,* 1) satisfies the conditions (i), (ii) and (iii) but it does not hold (iv) since (*b* ∗ *c*) ∗ 1 = *a* ≠ *b* = *b* ∗ (1 ∗ (1 ∗ *c*))

**Theorem 3.3**. If (X, ∗, 1) is a BS-algebra, then prove that y ∗ z = y ∗ (1 ∗ (1 ∗ z)) for all y, z ∈ X

**Proof*.***This comes from the condition *x* ∗ 1 = *x* and (*x* ∗ *y*) ∗ *z* = *x* ∗ (*z* ∗ (1 ∗ *y*)) Now, *y* ∗ *z* = (*y* ∗ *z*) ∗ 1 (by (ii))

 = *y* ∗ (1 ∗ (1 ∗ *z*)) (by (iv))

**Theorem 3.4**. If (X, ∗, 1) is a BS-algebra, then prove that (x ∗ y) ∗ (1 ∗ y) = x

for all x, y ∈ X

**Proof.**From (iv) with *z* = 1 ∗ *y* we have (*x* ∗ *y*) ∗ (1 ∗ *y*) = *x* ∗ ((1 ∗ *y*) ∗ (1 ∗ *y*)) From condition (i) (*x* ∗ *y*) ∗ (1 ∗ *y*) = *x* ∗ 1

From condition (ii),it becomes (*x* ∗ *y*) ∗ (1 ∗ *y*) = *x*

**Theorem 3.5.** If (X, ∗, 1) is a BS-algebra, then prove that x ∗ z = y ∗ z ⇒ x = y for all x, y, z ∈ X

**Proof.**If *x* ∗ *z* = *y* ∗ *z*,then (*x* ∗ *z*) ∗ (1 ∗ *z*) = (*y* ∗ *z*) ∗ (1 ∗ *z*)

 and by previous theorem, we get *x* = *y*

**Theorem 3.6.**  If (X, ∗, 1) is a BS-algebra, then prove that x ∗ (y ∗ z) = (x ∗ (1 ∗ z)) ∗ y

for all x, y, z ∈ X

**Proof**. Using (iv) we obtain (*x* ∗ (1 ∗ *z*)) ∗ *y* = *x* ∗ (*y* ∗ (1 ∗ (1 ∗ *z*)))

 = *x* ∗ (*y* ∗ *z*) (by thm(3.3))

**Theorem 3.7**. Let (X, ∗, 1) be a BS-algebra.Then prove that for all x, y ∈ X

1. x ∗ y = 1 ⇒ x = y

(ii)1 ∗ x = 1 ∗ y ⇒ x =y (iii)1 ∗ (1 ∗ x) = x

**Proof**. (i) Since *x* ∗ *y* = 1 ⇒ *x* ∗ *y* = *y* ∗ *y*, by theorem (3.5), we get *x* = *y*

1. If 1 ∗ *x* = 1 ∗ *y*, then 1 = *x* ∗ *x* = (*x* ∗ *x*) ∗ 1

 = *x* ∗ (1 ∗ (1 ∗ *x*))

 = *x* ∗ (1 ∗ (1 ∗ *y*))

 = (*x* ∗ *y*) ∗ 1

 1 = *x* ∗ *y*

By (i), *x* = *y*

1. For all *x* ∈ *X*, we get 1 ∗ *x* = (1 ∗ *x*) ∗ 1 (*by* (*ii*))

 = 1 ∗ (1 ∗ (1 ∗ *x*)) (*by* (*iv*))

By (ii) part of this theorem, we have *x* = 1 ∗ (1 ∗ *x*)

**Theorem 3.8.** If (X, ∗, 1) is a BS-algebra, then prove that (x ∗ y) ∗ y = x ∗ y2

for all x, y ∈ X

**Proof**.From (iv), We have (*x* ∗ *y*) ∗ *y* = *x* ∗ (*y* ∗ (1 ∗ *y*))

= *x* ∗ (*y* ∗ *y*)

= *x* ∗ *y*2

**Theorem 3.9.** If (X, ∗, 1) is a BS-algebra, then prove that (1 ∗ y) ∗ (x ∗ y) = x

for all x, y ∈ X

**Proof.** From theorem (3.6), (1 ∗ *y*) ∗ (*x* ∗ *y*) = ((1 ∗ *y*) ∗ (1 ∗ *y*)) ∗ *x*

 = 1 ∗ *x*

 = *x*

**Definition 3.10.** A BS-algebra (*X,* ∗*,* 1) is said to be commutative if

*x* ∗ (1 ∗ *y*) = *y* ∗ (1 ∗ *x*) for all *x, y* ∈ *X*

**Note 3.11**. The BS-algebra in example:3.2 (i) is commutative but the algebra in example:3.2 (iii)is not commutative since

*c* ∗ (1 ∗ *d*) = *b≠a* = *d* ∗ (1 ∗ *c*)

**Theorem 3.12.** If (X, ∗, 1) is commutative, then prove that (1 ∗ x) ∗ (1 ∗ y) = y ∗ x

for all x, y ∈ X

**Proof.**Since (*X,* ∗*,* 1) is commutative, then (1 ∗ *x*) ∗ (1 ∗ *y*) = *y* ∗ (1 ∗ (1 ∗ *x*))

 = *y* ∗ *x* (by thm(3.3))

**Theorem 3.13.** If (X, ∗, 1) is commutative, then prove that x ∗ (x ∗ y) = y for all

x, y ∈ X

**Proof.** By theorem (3.6),

Now*, x* ∗ (*x* ∗ *y*) = (*x* ∗ (1 ∗ *y*)) ∗ *x*

 = (*y* ∗ (1 ∗ *x*)) ∗ *x* (since (*X,* ∗*,* 1) is commutative)

 = *y* ∗ (*x* ∗ *x*)

 = *y* ∗ 1

 = *y*

**Corollary 3.14.** If (X, ∗, 1) is commutative, then the left cancellation law holds

(i.e) (x ∗ y) = x ∗ y′ ⟹ y = y′

**Proof*.***From theorem 3.13, we have *y* = *x* ∗ (*x* ∗ *y*) = *x* ∗ (*x* ∗ *y*′) = *y*′

**Theorem 3.15.** If (X, ∗, 1) is commutative, then prove that (1 ∗ x) ∗ (x ∗ y) = y ∗ x2

for all x, y ∈ X

**Proof.**

Now, (1 ∗ *x*) ∗ (*x* ∗ *y*) = ((1 ∗ *x*) ∗ (1 ∗ *y*)) ∗ *x* (by thm (3.6))

= (*y* ∗ *x*) ∗ *x* (by thm (3.12))

= *y* ∗ *x*2 (by thm (3.8))

# **4. Fuzzy Ideal**

**Definition 4.1**. A fuzzy set A in BS-Algebra X is called a fuzzy ideal of BS-Algebra X if it satisfies the following conditions

i) µ(1) ≥ µ(y)

ii) µ(y) ≥ min{µ(y ∗ x), µ(x)}for all x,y ∈ X

**Example 4.2**. A set *X* = {1*, a, b, c*} which satisfies the following table

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| \* | 1 | a | b | c |
| 1 | 1 | a | b | c |
| a | a | 1 | c | b |
| b | b | c | 1 | a |
| c | c | b | a | 1 |

Then (*X,* ∗*,* 1) is a BS-algebra. Define a fuzzy set *µ* in *X* by *µ*(1) = *µ*(*b*) = 0*.*7 and

*µ*(*a*) =*µ*(*c*) = 0*.*3 is the fuzzy ideal of *X*.

**Theorem 4.3.** If a fuzzy set µ in BS-algebra X is a fuzzy ideal, then

for all x ∈ X, µ(1) ≥ µ(x)

**Proof.** Straight forward

**Theorem 4.4.** If µ1 and µ2 be the two fuzzy ideals of BS-algebra X, then

µ1 ∩ µ2 is also a fuzzy ideal of BS-algebra X.

**Proof.**

Now, (*µ*1 ∩ *µ*2)(1) = (*µ*1 ∩ *µ*2)(*x* ∗ *x*)(by (i))

≥ *min*{(*µ*1 ∩ *µ*2)(*x*)*,* (*µ*1 ∩ *µ*2)(*x*)}

= (*µ*1 ∩ *µ*2)(*x*)

Therefore, (*µ*1 ∩ *µ*2)(1) ≥ (*µ*1 ∩ *µ*2)(*x*)

Also, (*µ*1 ∩ *µ*2)(*y*) = *min*{*µ*1(*y*)*, µ*2(*y*)}

≥ *min*{*min*{*µ*1(*x*)*, µ*1(*y*∗ *x*)}*, min*{*µ*2(*x*)*, µ*2(*y* ∗ *x*)}}

= *min*{*min*{*µ*1(*x*)*, µ*2(*x*)}*, min*{*µ*1(*y* ∗ *x*)*, µ*2(*y* ∗ *x*)}}

= *min*{(*µ*1 ∩ *µ*2)(*x*)*,* (*µ*1 ∩ *µ*2)(*y* ∗ *x*)}

(*µ*1 ∩ *µ*2)(*y*) ≥ *min*{(*µ*1 ∩ *µ*2)(*x*)*,* (*µ*1 ∩ *µ*2)(*y* ∗ *x*)}

Hence *µ*1 ∩ *µ*2 is a fuzzy ideal of *X*.

**Theorem 4.5.** Let µ be a fuzzy ideals of BS-algebra X. If x ∗ y ≤ z, then µ(x) ≥ min{µ(y), µ(z)}

**Proof.**Let *x, y, z* ∈ *X* such that *x* ∗ *y* ≤ *z*. Then (*x* ∗ *y*) ∗ *z* = 1,

*µ*(*x*) ≥ *min*{*µ*(*x* ∗ *y*)*, µ*(*y*)}

≥ *min*{*min*{*µ*((*x* ∗ *y*) ∗ *z*)*, µ*(*z*)}*, µ*(*y*)}

= *min*{*min*{*µ*(1)*, µ*(*z*)}*, µ*(*y*)}

= *min*{*µ*(*z*)*, µ*(*y*)}

Therefore, *µ*(*x*) ≥ *min*{*µ*(*z*)*, µ*(*y*)}

**Theorem 4.6.** Let µ be a fuzzy ideals of a BS-algebras X. If x ≤ y, then µ(x) ≥ µ(y)

(i.e) order reversing

**Proof.** Let *x, y* ∈ *X* such that *x* ≤ *y*. Then *x* ∗ *y* = 1

*µ*(*x*) ≥ *min*{*µ*(*x* ∗ *y*)*, µ*(*y*)}(by (ii) in def 4.1)

= *min*{*µ*(1)*, µ*(*y*)}

= *µ*(*y*) *µ*(*x*) ≥ *µ*(*y*)

Hence, *µ* is order reversing.

**Theorem 4.7.** Let µ be a fuzzy ideal of BS-algebras X. Then (...((x ∗ a1) ∗ a2) ∗ an = 0 for any x, a1, a2...an ∈ X, implies µ(x) ≥ min{µ(a1), µ(a2)...µ(an)}

**Proof.**Using induction on *n* and by previous two theorems we can easily prove the theorem

**Theorem 4.8.** Let B be a crisp subset of BS-algebras X. Suppose that A = µ(x) is a fuzzy set in X defined by µ(x) = λ if x ∈ B and µ(x) = τ if x ∉ B for all λ, τ ∈ [0, 1] with λ ≥ τ. Then A is a fuzzy ideal of X if and only if B is a ideal of X

**Proof.**Assume that *A* is a fuzzy ideal of *X*. Let *x* ∈ *B*.

Let *x, y* ∈ *X* be such that *y* ∗ *x* ∈ *B* and *x* ∈ *B*. Then *µ*(*y* ∗ *x*) = *λ* = *µ*(*x*),

and hence *µ*(*y*) ≥ *min*{*µ*(*x*)*, µ*(*y* ∗ *x*)} = *λ.* Thus *µ*(*y*) = *λ* (i.e) *y* ∈ *B.*

Therefore *B* is a ideal of *X*.

Conversely, assume that *B* is an ideal of *X*. Let *y* ∈*X.*

Let *x, y* ∈ *X*. If *y* ∗ *x* ∈ *B* and *x* ∈ *B*, then *y* ∈ *B.*

Hence, *µ*(*y*) = *λ* = *min*{*µ*(*y* ∗ *x*)*, µ*(*x*)}

If *y* ∗ *x ∉ B* and *x ∉ B*, then clearly *µ*(*y*) ≥ *min*{*µ*(*x*)*, µ*(*y* ∗ *x*)}

If exactly one of *y* ∗ *x* and *x* belong to *B*, then exactly one of *µ*(*y* ∗ *x*)*, µ*(*x*) is equal to *τ* . Therefore, *µ*(*y*) ≥ *τ* = *min*{*µ*(*x*)*, µ*(*y* ∗ *x*)}

Consequently, *A* is a fuzzy ideal of *BS-algebras X*.

**Theorem 4.9.** A fuzzy set µ is an ideal of BS-algebras X then the set U (µ : t) is an ideal of X for every t ∈ [0, 1]

**Proof.** Suppose that *µ* is an fuzzy ideal of *X*. For *t* ∈ [0*,* 1]. Let *x, y* ∈ *X* be such that *y* ∗ *x* ∈ *U* (*µ* : *t*) and *x* ∈ *U* (*µ* : *t*). Then *µ*(*y*) ≥ *min*{*µ*(*x*)*, µ*(*y* ∗ *x*)}.

Then *y* ∈ *U* (*µ* : *t*). Hence *U* (*µ* : *t*) is an ideal of *X*.

**Definition 4.10.** Let the two BS-algebras be X and Y. Let f be a mapping from the X to Y. Let B be a fuzzy set in Y . Then the inverse image of B is defined as

f −1(µ)(x) = µ(f (x)). The set *f* −1(*B*) = {*f* −1(*µ*)(*x*) : *x* ∈ *X*} *is a fuzzy set.*

**Theorem 4.11.** Let f : X → Y be a homomorphism of BS-algebras. If B is a fuzzy ideal of Y, then the pre-image f −1(B) of B under f in X is a fuzzy ideal of X

**Proof**. For all *x* ∈ *X*, *f* −1(*µ*)(*x*) = *µ*(*f* (*x*)) ≤ *µ*(1) = *µ*(*f* (1)) = *f* −1(*µ*)(1) Therefore, *f* −1(*µ*)(*x*) ≤ *f* −1(*µ*)(1)

Let *x, y* ∈ *X*. Then *f* −1(*µ*)(*x*) = *µ*(*f* (*x*))

≥ *min*{*µ*(*f* (*x*) ∗ *f* (*y*))*, µ*(*f* (*y*))}

≥ *min*{*µ*(*f* (*x* ∗ *y*))*, µ*(*f* (*y*))}

= *min*{*f* −1(*µ*)(*x* ∗ *y*)*, f* −1(*µ*)(*y*)}

Therefore, *f* −1(*µ*)(*x*) ≥ *min*{*f* −1(*µ*)(*x* ∗ *y*)*, f* −1(*µ*)(*y*)}

Hence *f* −1(*B*) = {*f* −1(*µ*)(*x*) : *x* ∈ *X*} is a fuzzy ideal of *X*.

**Theorem 4.12**. Let f : X → Y be an onto homomorphism of BS-algebra. Then B is a fuzzy ideal of Y , if f −1(B) of B under f in X is a fuzzy ideal of X

**Proof.** For any *u* ∈ *Y* , there exists *x* ∈ *X* such that *f* (*x*) = *u*

Then *µ*(*u*) = *µ*(*f* (*x*)) = *f* −1(*µ*)(*x*) ≤ *f* −1(*µ*)(1) = *µ*(*f* (1)) = *µ*(1)

Therefore, *µ*(*u*) ≤ *µ*(1)

Let *u, v* ∈ *Y* . Then *f* (*x*) = *u* and *f* (*y*) = *v* for some *x, y* ∈ *X*.

Thus, *µ*(*u*) = *µ*(*f* (*x*)) = *f* −1(*µ*)(*x*)

≥ *min*{*f* −1(*µ*)(*x* ∗ *y*)*, f* −1(*µ*)(*y*)}

= *min*{*µ*(*f* (*x* ∗ *y*))*, µ*(*f* (*y*))}

= *min*{*µ*(*f* (*x*) ∗ *f* (*y*))*, µ*(*f* (*y*))}

= *min*{*µ*(*u* ∗ *v*)*, µ*(*v*)}

Therefore, *µ*(*u*) ≥ *min*{*µ*(*u* ∗ *v*)*, µ*(*v*)}

Then *B* is a fuzzy ideal of *Y*

**Theorem 4.13.** Let A and B be fuzzy ideals of X, then A × B is a fuzzy ideal of X × X

**Proof**. For any (*x, y*) ∈ *X* × *X*, we have

(*µ*1 × *µ*2)(1*,* 1) = *min*{*µ*1(1)*, µ*2(1)}

≥ *min*{*µ*1(*x*)*, µ*2(*y*)} for all *x, y* ∈ *X*

= (*µ*1 × *µ*2)(*x, y*)

Therefore, (*µ*1 × *µ*2)(1*,* 1) ≥ (*µ*1 × *µ*2)(*x, y*)

Let (*x*1*, y*1)*,* (*x*2*, y*2) ∈ *X* × *X*. Then

(*µ*1 × *µ*2)(*x*1*, y*1) = *min*{*µ*1(*x*1)*, µ*2(*y*1)}

≥ *min*{*min*{*µ*1(*x*1 ∗ *x*2)*, µ*1(*x*2)}*, min*{*µ*2(*y*1 ∗ *y*2)*, µ*2(*y*2)}}

= *min*{*min*{*µ*1(*x*1 ∗ *x*2)*, µ*2(*y*1 ∗ *y*2)}*, min*{*µ*1(*x*2)*, µ*2(*y*2)}}

= *min*{(*µ*1 × *µ*2)((*x*1 ∗ *x*2)*,* (*y*1 ∗ *y*2))*,* (*µ*1 × *µ*2)(*x*2*, y*2)}

Therefore, (*µ*1 × *µ*2)(*x*1*, y*1) ≥ *min*{(*µ*1 × *µ*2)((*x*1 ∗ *x*2)*,* (*y*1 ∗ *y*2))*,* (*µ*1 × *µ*2)(*x*2*, y*2)}

Hence, *A* × *B* is a fuzzy ideal of *X* × *X*

**Theorem 4.14.** Let A and B be fuzzy sets in X such that A × B is a fuzzy ideal of

X × X, then

1. Either µ1(1) ≥ µ1(x) or µ2(1) ≥ µ2(x) ∀ x ∈ X
2. If µ1(1) ≥ µ1(x) ∀ x ∈ X, then either µ2(1) ≥ µ1(x) or µ2(1) ≥ µ2(x)
3. If µ2(1) ≥ µ2(x) ∀ x ∈ X, then either µ1(1) ≥ µ1(x) or µ1(1) ≥ µ2(x)

**Proof.** (i) Assume that *µ*1(*x*) *> µ*1(1) and *µ*2(*y*) *> µ*2(1) for some *x, y* ∈ *X*.

Then (*µ*1 × *µ*2)(*x, y*) = *min*{*µ*1(*x*)*, µ*2(*y*)}

 *> min*{*µ*1(1)*, µ*2(1)}

 = (*µ*1 × *µ*2)(1*,* 1)

 ⟹ (*µ*1 × *µ*2)(*x, y*) *>* (*µ*1 × *µ*2)(1*,* 1) ∀ *x, y* ∈ *X,* which is a contradiction*.*

Hence (i) is proved.

1. Again assume that *µ*2(1) *< µ*1(*x*) and *µ*2(1) *< µ*2(*y*) ∀ *x, y* ∈ *X*

Then(*µ*1 × *µ*2)(1*,* 1) = *min*{*µ*1(1)*, µ*2(1)}

= *µ*2(1)

Now,(*µ*1 × *µ*2)(*x, y*) = *min*{*µ*1(*x*)*, µ*2(*y*)}

*> µ*2(1)

= (*µ*1 × *µ*2)(1*,* 1)*,* which is a contradiction*.*

Hence (ii) is proved

1. The proof is similar to (ii)

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