**A Deterministic Inventory Model for Deteriorating Items with a Generalized Exponential Decreasing Demand, Constant Holding Cost, and Constant Deterioration Rate**

**S.Sobia**, Department of Mathematics, Chikkaiah Naicker College, Erode Tamilnadu, India. email: ssobia.cnc@gmail.com

**M.Pradeepa,** Department of Mathematics, Chikkaiah Naicker College, Erode Tamilnadu, India. email: pradeepa.cnc@gmail.com

**Dr.M.Valliathal,** Department of Mathematics, Chikkaiah Naicker College, Erode Tamilnadu, India. email: balbal\_ba@yahoo.com

**Abstract**

In this research paper, an EOQ model is developed for deteriorating items with generalized exponential decreasing demand, by considering holding cost and deteriorating rate as constant. An analytical solution is determined to maximize the total profit. The proposed model considered here does not allow shortages. The implementation of the model is illustrated by a few numerical examples. Sensitivity Analysis is performed to portray the effect of changes in the parameters of the optimum solution.

**Keywords:** Inventory, Demand, Profit, Holding cost, Deterioration rate.

1. **Introduction**

Inventories are vital to any business or enterprise, and it is very essential for the smooth functioning and efficient working of an organization. Inventory consists of resources which are usable but idle stock for some point of time. Inventory may be raw materials used for production of goods, semi-finished products, finished products, packaging, spares and others stocked in order to meet the future demand. The main objective of an enterprise is to decide how much to order, when to order and how much to stock to minimize the total cost or to maximize the total profit.

 Generally, deterioration is defined as the decay, damage, spoilage, evaporation and obsolescence of stored items and it results in decreasing usefulness. For example, vegetables, fruits, food items, perfumes, chemicals, pharmaceuticals, drugs, radioactive substances, electric equipment’s, etc. In formulating inventory models, the deterioration of items should not be ignored. Therefore, the effect of deterioration on these items must be taken into account in the inventory system.

1. **Literature review**

 Many researchers have studied the production and inventory problems of deteriorating items widely. Inventory of deteriorating items was initially studied by Whitin [1]. He considered the deterioration of fashion goods at the end of prescribed storage period. Following his work Ghare P.M & Schrader G. F [2] were the two earliest researchers to consider continuously decaying inventory for a constant demand. Wee H.M [3] presented an EOQ model for deteriorating items with partial back ordering. Shah and Jaiswal [4] developed an order-level inventory model for decaying items with deterioration rate as constant. By considering the demand as a linear function of time Dave and Patel [5] gave a deteriorating inventory model. Dash B.P, Singh T and Pattanyak [6] proposed an inventory model for deteriorating inventory items with exponential declining demand and time varying holding cost. Giri and Chaudhuri [7] developed a deterministic inventory model for deteriorating items with non-linear holding cost and stock dependent demand rate. Chang and Dye [8] proposed and EOQ model for deteriorating items with time varying demand and partial backlogging. They also proposed an inventory model for perishable items with permissible delay in payments and shortages. Lin et.al [9] proposed an EOQ model for deteriorating items with time varying demand and allowing shortages. Papachristos and Skouri [10] developed an optimal replenishment policy for deteriorating items with exponential type backlogging rate and time varying demand. They also presented a continuous review inventory model for deteriorating items with time dependent demand and allowing shortages. S.K Goyal and B.C Giri [11] developed a model by considering recent trends in modelling of deteriorating inventory. Wu [12] proposed an EOQ model for Weibull deteriorating items with time varying demand and allowing shortages. Dye and Ouyard [13] developed an EOQ model for perishable products with stock dependent selling rate and shortages allowed. Hou & Lin [14] developed an EOQ model for deteriorating items with price and stock dependent selling rate. Maragatham and Palani [15] developed a model for perishable items with lead time price dependent demand and allowing shortages. I. Aliya and B. Sani [16] developed an inventory model for deteriorating items with a generalized exponential increasing demand, constant holding cost and constant deterioration rate.

 In this paper we have developed a deterministic inventory model for deteriorating items with generalized exponential decreasing demand by considering holding cost and deterioration rate as constant.

1. **Notations and Assumptions**

Mathematical formulation of the model is based on the following assumptions and notations

* 1. **Notations**

 : Ordering cost per order

 : Level of inventory at any time ,

 : The exponential demand rate where and are constants

 : Constant deterioration rate

 : Constant holding cost i.e., , Where *i* is the inventory carrying charge and is the unit cost of an item.

 : Ordering cycle length

 : Initial stock

 : Total profit per unit time

 : Optimal length of the cycle

 : The economic order Quantity

 : Selling price per unit

 : Maximum profit per unit time

* 1. **Assumptions**
* The demand rate is deterministic and is a generalized exponential decreasing function of time
* Holding cost is considered to be constant
* Deterioration rate is assumed to be constant
* Shortages are not allowed
* There is no lead time and rate of replenishment is infinite
* The time horizon is infinite
* The model is developed for single item inventory only
1. **Mathematical formulation and solution of the model**

Consider an inventory system in which the inventory level is maximum at and the inventory level gradually decreases due to demand and deterioration and it reaches zero level at .

The differential equation describing the inventory level at any time in the interval is given by

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

The solution of equation (1) is

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

Using the boundary condition when in equation (2) gives

|  |  |  |
| --- | --- | --- |
|  |  |  |

Substituting in equation (2), we get

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | (3) |

 The initial order quantity is obtained by putting the boundary condition in equation (3) as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

The total demand during the cycle period is

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

Number of deteriorated units = Initial order quantity- Total demand in the cycle period

 =

 =

 =

Deterioration cost for the cycle period is

 = \*(The number of deteriorated units)

 =

Total inventory holding cost for the cycle [0,T] is

 IHC =

 =

 =

Total Profit per unit time is

 [Sales revenue- Purchase cost-Ordering cost- Deterioration Cost – Inventory holding cost]

To find maximum profit per unit time

Equating the above equation to zero and simplifying by multiplying both sides by

 in order to determine that maximizes the total profit per unit time as follows.

The value of T obtained gives the maximum profit provided it satisfies Now

Substituting the value of in we see that which shows that the total profit function we obtained give the maximum value.

1. **Numerical example**

In this section we give numerical example to illustrate the model.

**Example 1.**

For the developed model, the value of various parameter can be taken as follows.

Rs 1000/order,, , , , Rs 150/unit, P=350/unit, .

Substituting these values in (9), we obtain optimum length of cycle Substituting the value of in (8), we get the maximum profit per unit time On substituting the value of in (4) gives the economic order quantity.

 The value of satisfies as already mentioned.

Example 2.

 Using the same values as in example 1 with a changed as 2, the solutions are T=0.204608(75days) ,TP=285812.24 and Q=305.1736.

Example 3.

 Using the same values as in example 1 with a changed as 3, the solutions are T=0.124349(45days) ,TP=787362.51 and Q=502.3295.

The above values for different values of are tabulated as follows

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1 | 0.336237(123days) | 102807.85 | 185.5914 |
| 2 | 0.204608(75days) | 285812.24 | 305.1736 |
| 3 | 0.124348(45days) | 787362.51 | 502.3295 |

1. **SENSITIVITY ANALYSIS**

 Sensitivity analysis is performed on example 1. By changing each of the parameter and by 40%, 20%, -20%, -40%, while keeping the remaining parameter at their original values. The corresponding changes in the cycle time, maximum profit per unit time, and EOQ are shown in the table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Parameter | % change in parameter |  |  |  |
|  | 2040-20-40 | 0.397236(150days)0.368037(134days)0.301004(110days)0.260938(95days) | 101717.22102239.98103435.54104147.33 | 219.8666203.4364165.8795143.5393 |
|  |  4020-20-40 | 0.284539(104days)0.307164(112days)0.375556(137days)0.433032(158days) | 145219.63123991.1181684.3460644.92 | 219.3648203.1854166.1304144.0410 |
|  | 4020-20-40 | 0.307542(112days)0.320959(117days)0.353850(129days)0.374456(137days) | 102265.71102531.12103097054103402.25 | 170.5847177.6082194.7749205.4929 |
|  | 4020-20-40 | 0.332499(121days)0.334353(122days)0.338154(123days)0.340103(124days) | 102737.79102772.72102843.19102878.74 | 183.3747184.4735186.7289187.8865 |
|  | 4020-20-40 | 0.308736(113days)0.321605(117days)0.353076(129days)0.372742(136days) | 102278.30102537.39103091035103389.96 | 170.2000177.3976195.0346206.0807 |
|  | 4020-20-40 | 0.288984(105days)0.309931(113days)0.370671(135days)0.418542(153days) | 69216.7685993.81119669.38136595.05 | 159.1692170.8680204.9170231.8831 |
|  | 4020-20-40 | 0.329298(120days)0.332714(121days)0.339873(124days)0.343629(125days) | 178793.26140800.2364816.1626825.16 | 181.7047183.6177187.6293189.7346 |

The following observations are made on the basis of the above table.

1. The increase in the value of the parameter Oc results in the increase in the values of T and Q and decrease in the value of TP.
2. The increase in the value of the parameter m results in the increase in the value of Q and decrease in the values of T and TP.
3. The increase in the value of the parameter θ1 results in the decrease in the values of T, TP and Q.
4. The increase in the value of the parameter b results in the decrease in the values of T, TP and Q.
5. The increase in the value of the parameter i results in the decrease in the values of T, TP and Q.
6. The increase in the value of the parameter c results in the decrease in the values of T, TP and Q.
7. The increase in the value of the parameter p results in the increase in the values of TP and decrease in the values of T and Q.
8. **CONCLUSION**

This paper presents deterministic inventory model for deteriorating items with exponential decrease in demands. Commodities like fruits, vegetables, fashion items and computer chips etc. are deteriorating in nature and have fixed life and devolve with time. This model is solved to maximize the profit. Numerical examples were given to illustrate the model. The model can be extended for deteriorating items having linear and quadratic decrease in demand, price dependent demand etc.

**References:**

1. T.M. Whitin (1957), Theory of inventory management, Princeton University Press, Princeton, NJ.
2. P.M. Ghare and G.F. Schrader (1963), A Model for an exponential decaying inventory, Journal of industrial engineering, 14, 238-243.
3. H.M. Wee (1993), Economic order quantity model for deteriorating items with partial backordering, Computer and industrial engineering, 24, 449-458.
4. Y.K. Shah and M.C. Jaiswal (1977), An Order-Level inventory model for a system with constant rate of deterioration, Opsearch, 14, 174-184.
5. U.Dave and L.K.Patel (1981), Policy inventory model for deteriorating items with time proportional demand, Journal of the operational research society, 32, 137-142.
6. B.P. Dash, T. Singh and H. Pattnayak (2014), An inventory model for deteriorating items with exponential declining demand and time-varying holding cost, American journal of operations research.
7. B.C.Giri and K.S. Chaudhuri (1998), A Deterministic inventory model for deteriorating items with stock dependent demand rate and non-linear holding cost, European journal of operational research, 105, 464-467.
8. H.J Chang and C.Y. Dye (1999), An EOQ model for deteriorating items with time varying demand and partial backlogging, Journal of operational research society, 50, 1176-1182.
9. B. Lin, B. Tan and W.C. Lee (2000), An EOQ model for deteriorating items with time varying demand and shortages, International journal of systems science, 31, 391-400.
10. S. Papachristas and K. Skouri (2000), An optimal replenish policy for an inventory model of deteriorating items with time varying demand and exponential type backlogging rate, Operation research letters, 27, 175-184.
11. S.K. Goyal and B.C. Giri (2001), Recent trends in modelling of deteriorating inventory, European journal of operation research, 134, 1-16.
12. K.S. Wu (2002), An EOQ model for Weibull deteriorating items with time varying demand and partial backlogging, International journal of systems science, 33, 323-329.
13. C.Y. Dye and L.Y. Ouyang (2005), An EOQ model for perishable items under stock-dependent selling rate and time- dependent partial backlogging, European journal of operational research, 163(3), 776-783.
14. K.L. Hou and L.C. Lin (2006), An EOQ model for deteriorating items with price and stock dependent selling rate under inflation and time value of money, International journal of systems science, 37, 9-13.\
15. M. Maragatham and R. Palani (2017), An inventory model for deteriorating items with lead time, price dependent demand and shortages, Advances in computational science and technology, 10(6), 1839-1847.
16. I. Aliya and B. Sani (2020), An inventory model for deteriorating items with a generalized exponential increasing demand, constant holding cost and constant deterioration rate, Applied mathematical sciences, 14(15), 725-736.