##### Banach Fixed Contraction Mapping Theorem in Vector -metric Spaces

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**Abstract**

We demonstrate the Banach contraction mapping theorem on vector -metric space. We also give an example to explain our results.

**Keywords:** Vector metric space, Vector lattice, Vector -metric space.

## 1 Introduction

Banach Contraction Principle(BCP) was demonsted firstly by S. Banach [2] in 1922. It has a vital role in fixed point(FP) theory and became very famous due to iterations. Many researchers are establishing new results in various generalizations of metric spaces. -metric space is one of the generalizations in metric spaces. In 2012, -metric space was defined by Sedghi et al.[7]. We start with some definitions and results for vector -metric spaces(VSMS).

**Definition 1.**[4] On a set , a relation is a partial order if it follows the conditions stated below:

(1) (reflexive)

(2) and implies

(3) and implies

.

The set with partial order is known as partially ordered set (poset).

A partially ordered set is called linearly ordered if for , we have either or .

**Definition 2.**[4] Let be linear space which is real and be a poset . Then the poset is said to be an ordered linear space if it follows the properties mentioned below:

(1)

(2)

and .

**Definition 3.**[4] A poset is called lattice if each set with two elements has an infimum and a supremum.

**Definition 4.**[4] An ordered linear space where the ordering is lattice is called vector lattice(VL).

**Definition 5.**[4] A VL is called Archimedean if for every where

**Definition 6.**[3] Let be VL and be a nonvoid set. A function is called vector metric on if it follows the conditions stated below:

(1) iff

(2)

The triple is called vector metric space.

Now, vector valued -metric space is defined as follows:

**Definition 8.** [10} Let be VL and be a nonvoid set. A function is called vector -metric on that satisfies the conditions mentioned below:

(1) ,

(2) iff ,

(3) +

for all .

The triplet is called vector -metric space(VSMS).

**Example 1**  *Let be a nonvoid set and be a VL. A function is defined by*

then the triplet is VSMS.

**Definition 9.** A sequence in VSMS is called -convergent to some if there is a sequence in satisfying and and denote it by .

**Definition 10.** A sequence in VSMS is known as -Cauchy sequence if there is a sequence in satisfying and holds for all and .

**Definition 11.** If each -Cauchy sequence in is -converges to a limit in then VSMS is called -complete .

**Lemma**[8] For VSMS ,

## 2 Main Results

**Theorem 1** *Let be a VSMS which is K-complete and be Archimedean. Suppose the transformation satisfies*

where . Then has FP in which is unique and for any , iterative sequence defined by , for all , -converges to FP of .

**Proof** Let and defined by for .Then we have

Thus for

Thus is a -Cauchy sequence because be Archimedean. Then by -completeness of , there exist such that . So there exist in such that and . Since

then , i.e. .

We can also verify the following theorem as above.

**Theorem 2** *Let be a VSMS which is complete and be Archimedean. Suppose the transformation satisfies*

for all , where and are positive and . Then has FP in and for any , iterative sequence defined by , , -converges to FP of .

**Example 2** *Let with coordinatewise ordering and let*

The mapping is defined by

Then is VSMS which is complete.

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