**KAMAL TRANSFORM FOR ADDRESSING THE PROBLEM OF NON-LINEAR VOLTERRA INTEGRAL EQUATION OF SECOND KIND**

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**ABSTRACT:** Solving the second-kind nonlinear Volterra integral equation is the aim of this effort. The Kamal transform was used to find the compact form solution of the nonlinear Volterra integral problem of second class. A pair of numerical issues was examined, and the Kamal transform was utilized to ascertain their precise solutions. The investigation's conclusions show that the Kamal transform adequately solved the study's challenge. The Kamal transform tackles the nonlinearity and complexity of Volterra integral equations of the second kind, paving the path for practical solutions across multiple fields.

**KEYWORDS:** Kamal Transform; Inverse Kamal Transform; Convolution; Volterra Integral Equation; Dirac Delta Function.

**MATHEMATICS SUBJECT CLASSIFICATION:** 35A22; 44A10; 44A35; 45D05; 45G10

**1.INTRODUCTION:** Volterra integral equations are employed in many academic fields, including physics, biology, mechanics, and medicine, to explain a wide range of real-world issues [1-3]. Various researchers solved Volterra integral equations (linear and non-linear) by using numerical [4-7] and analytical methods [8-15] in their research. The kind of the Volterra integral equations determines which of these approaches to use. Due to the non-linearity of the unknown functions, solving non-linear Volterra integral equations becomes increasingly difficult. Using a variety of integral transformations, researchers [16–22] have recently resolved various problems of mathematics, medical science, biology, chemistry, and mechanics. Aggarwal et al. [23] used Kamal transform and determined the complete solutions of problems of population growth and decay. Aggarwal et al. [24] studied Kamal transform in detail and organized the results of the duality of this transform with other transforms in the table.

In the current study, the second-kind nonlinear Volterra integral problem is solved using the Kamal transform. We are able to get the required solution for our non-linear problem by applying this integral transform to change the issue into an algebraic equation that is easily solved using traditional techniques.

**2. SYMBOLS NOMENCLATURE**

, Kamal transform operator;

, inverse Kamal transform operator;

, the set of natural numbers;

, belongs to;

, the usual factorial notation;

, the classical Gamma function;

, the set of real numbers

**3. DEFINITION OF KAMAL TRANSFORM**

A sectionally continuous exponential order function , has the Kamal transform and it is given by [10]

(1)

**4. INVERSE** **KAMAL TRANSFORM [14]**

The inverse Kamal transform of designated by, is another function having the property that

Tables 1-3 offer an overview of the Kamal transform's key operating properties, as well as the Kamal transforms and inverse Kamal transforms of a few simple functions.

**Table-1:** Some significant operational features of Kamal transform [14]

|  |  |  |
| --- | --- | --- |
| S.N. | Type of Characteristic | Mathematical Form |
| 1 | Linearity | , where are arbitrary constants |
| 2 | Change of Scale | If then |
| 3 | Translation | If then |
| 4 | Convolution | If and then |

**Table-2:** Kamal transforms of a few basic functions [23, 24]

|  |  |  |
| --- | --- | --- |
| S.N. |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |

**Table-3:** Inverse Kamal transforms of a few basic functions [23]

|  |  |  |
| --- | --- | --- |
| S.N. |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |

**5. MEAN VALUE THEOREM FOR INTEGRALS [25]**

If a function is continuous on , then there exists a number in such that

(2)

**6. DIRAC DELTA FUNCTION [26]**

The Dirac delta function is considered as the limiting form of the function

as .

**7. KAMAL TRANSFORM OF DIRAC DELTA FUNCTION**

If is a continuous function at , then

, by equation (2).

As , we have .

In particular, when , we have

.

**8. KAMAL TRANSFORM FOR HANDLING NON-LINEAR VOLTERRA INTEGRAL EQUATION OF SECOND KIND**

In this research, we assumed the nonlinear Volterra integral equation of second kind in the following form.

, (3)

where and are unknown and known functions respectively.

Operating Kamal transform on equation (3), we get

(4)

Use of convolution theorem of Kamal transform in equation (4) gives

(5)

After operating inverse Kamal transform on equation (5), the solutions of equation (3) are given by

(6)

**8. NUMERICAL PROBLEMS:** This section presents two numerical problems that serve as an example of how to solve the non-linear Volterra integral equation of second kind in detail.

**PROBLEM: 8.1** Take into account the following non linear second-kind Volterra integral equation

(7)

**SOLUTION:** Operating Kamal transform on equation (7), we get

(8)

Using convolution theorem of Kamal transform in equation (8), we have

and (9)

After operating inverse Kamal transform on equation (9), the solutions of equation (7) are given by

and

and

and .

**PROBLEM: 8.2** Take into account the following non linear second-kind Volterra integral equation

(10)

**SOLUTION:** Operating Kamal transform on equation (10), we get

(11)

Using convolution theorem of Kamal transform in equation (11), we have

and (12)

After operating inverse Kamal transform on equation (12), the solutions of equation (10) are given by

and

⇒ and

and .

**9. CONCLUSIONS:** Using the Kamal transform, the authors of the paper effortlessly solved the non-linear Volterra integral equation of second kind in compact form. According to the study's conclusions, the Kamal transform is a highly helpful integral transform that can be used to solve a non-linear Volterra integral equation of the second category in compact form with a minimal amount of time-consuming computation. The Kamal transform may be used in the years to come to tackle intricate scientific and technical issues that can be related to one or more non-linear Volterra integral equations of the second class.

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