Some new oscillation criteria for alpha-fractional differential equation

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 ABSTRACT

In this paper, we establish some new oscillation criteria for alpha-fractional differential equation. We obtained for the oscillation of all solutions by using the uniform Lipschitz condition and also the generalized Riccati transformation. Examples are provided that to illustrate our theoretical results.

Keywords—Oscillation, Alpha-fractional, nonlinear.

# INTRODUCTION

The theory of fractional differential equations is considered as an important tool in modeling real life phenomena. In 17th century, the nation of fractional differential derivative first appeared. It is well known that fractional differential equations are a more general form of the integer order differential equations, extending those equations to an arbitrary (non-integer) order. The definitions involve integration most frequently which is nonlocal: Riemann-Liouville derivative & Caputo derivative [1,2]. Those fractional derivatives are complicated and lack some basic properties, like the product rule and chain rule. Khalili [3], introduced a new fractional derivative called the conformable derivative or -fractional derivative which closely resembles the classical derivative [4-6] . They are extensively being used in electromagnetic field, biological, physics, electro chemistry, control theory, ecological, viscoelasticity [7-10].

 In the last few decades, there has been a lot of interest in deriving sufficient conditions for the oscillation and non-oscillation of solutions of classes of differential equations [11-16]. Oscillatory solution plays an active role in the quantitative and qualitative theory of mixed -fractional differential equations. Motivated by Nehari[17], we propose the following model of the form

here is a constant.

The following assumption:

 (A1)

The function is defined for ,

(A2)

(A3) is continuous in for fixed ;

(A4) In a neighborhood of every in satisfies a uniform Lipschitz condition,

(A5) is constant and is continuous function.

A solution of (1) is oscillatory if it has arbitrarily large zero, and otherwise it is non-oscillatory. Equation (1) said to be oscillatory if all their solutions are oscillatory.

This paper is organized as follows: We present the relevant definition and lemmas in Section II. In Section III we discuss the main result and finally we present the example to illustrate our theoretical results.

# Preliminaries

In this section, we give some basic definitions, integrals and lemmas which are useful throughout the paper

**A. Definition: 1**

Given Then, the conformable fractional derivative of of order is defined by

.

For every . If is -differentiable in some and exists, then we define

**Definition: 2**

where the integral is the usual Riemann improper integral and .

**B. Theorem: 1**

Let and be -differentiable at some point . Then,

1. ()
2. 0, c is constant,
3.
4. ,
5. If is differentiable, then

**C. Lemma: 1**

To prove that

Proof.

 .

Hence the proof.

**Lemma: 2**

To prove that .

Proof.

 .

Hence the proof.

# MAIN RESULT

# Oscillation with uniform Lipschitz condition

 In the following theorem, we establish some new oscillation withuniform Lipschitz condition and convex function

## **Theorem: 1**

Let be defined for and satisfy there the following conditions

#### is continuuous in for fixed ,

####  is neighborhood in every in satify a uniform Lipschitz condition,

#### For fixed , is a nondecreasing function of If is defined by

and if, for some positive and all positive is non-increasing for then equation (2) is non-oscillatory. This condition is the best possible in the sense that the conclusion does not hold for

Proof.

 If and are two consecutive zeros of a solution of (1).

Since,

,

using (4), we get

 is ( is fixed) a increasing function of By (2), is convex in , and we have

(

Hence,

,

where is arbitrary positive number and taking from to , using (4) we get

Our assumption imply that is decreasing in for fixed As a result, the partial derivative exists for almost all We introduce the function

and we use the Lemma 1 and Lemma 2, we get

Since, is decreasing for it follows from (8)

For is positive and almost all So, let , we get

We using (7) in (9),

Multiply by and taking between two consecutive zeros of and by (2), we have

From (6), we get

Here if are consecutive zeros of and using (11),

The convexity of , we obtain that

Here is any positive number, If we assume, then Since is increasing function of (2) become that So,

,

(13) become,

By assumption, is decreasing for .Hence, and (14) leads to

Thus,

and (12) yields,

for the number of zeros which a solution of (1), which vanishes at can have in any interval Hence, all solutions are non-oscillatory, the theorem is proved.

**Oscillation with Riccati transformation**

In the following theorem, we are using the Riccati techniques and Philo’s type to demonstrate the new oscillation.

## **Theorem: 2**

If there exists a function such that

 (15)

here Then every solution of (1) is oscillatory.

Proof.

Suppose that is a non-oscillatory solution of (1). We define the Riccati transformation,

,

Taking on both side

.

Taking , we get

which leads to contradictions (15).

In the sequel, we say that a function satisfying > 0 for where , , Furthermore, has continuous derivatives

## **Theorem: 3**

Suppose that there exists a function such that

Then every solution of (1) is oscillatory.

Proof.

Suppose that is a non-oscillatory solution of (1). Multiply by and taking in (16), we get

which leads to contradictions. Hence the proof.

## **Corollary: 1**

Assume that the conditions of Theorem 3 hold with (17) replaced by

 and

Then every solution of (1) is oscillatory.

# EXAMPLES

## **Example:1**

Consider the conformable differential equation,

here , and

 .

Hence, all the conditions of Theorem 2 are satisfied. Therefore, every solution of (18) is oscillatory. In fact, is one such solution of (18).

## **Example:2**

Consider the conformable differential equation,

here and

 .

 Hence, all the conditions of Theorem 2 are not satisfied. In fact, is a non-oscillatory solution of (19).

**CONCLUSION**

In this article, we have identified some new oscillation or non-oscillation criteria for alpha-fractional differential equation. Since the obtained results are general forms of earlier works they would help for the investigate in future studies. Examples are provided that to illustrate our theoretical results.

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