**Inverse Optimization for Mathematical Programming Problem: An Introduction**

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**Abstract**

An inverse optimization problem is to find the values of parameters (cost, capacity, travel time, etc.) which makes the given solution optimum and which are differ from the given parameters as little as possible. In this chapter, we have considered a mathematical programming problem and discussed its various inverse problems and their formulation, reported in the literature.

**Introduction**

A variety of real life problems can be formulated as a mathematical programming problem and solved by using suitable techniques. Whenever we model these problems mathematically, it is assumed that all the parameters associated with the problem are known exactly and we wish to find the solution which is optimal for the present values of parameter. However, in practice, there are many situations when we are not very much sure about these parameters or we only have some estimates of these parameters, but we may have a solution from the observation, experiment or experiences. The known solution may or may not be optimal for the present values of parameters, so we need to adjust these parameters to make the given solution optimal. This problem can be considered as an inverse problem, but whenever we talk about optimization, we always look for the best solution i.e. the adjustment of the parameters should be minimum or the cost associated behind it should be minimum. Thus, an inverse optimization problem is to find the values of parameters (cost, capacity, travel time, etc.) which makes the given solution optimum and which are differ from the given parameters as little as possible. The original problem is called the forward problem.

Thus in a forward problem we identifies the values of observable parameters (decision variables) given the values of the model parameters (cost coefficients, right-hand side vector, and the constraint matrix). An inverse optimization problem consists of inferring the values of the model parameters (cost coefficients, right-hand side vector, and the constraint matrix) given the values of observable parameters (decision variables). General speaking, an inverse optimization problem is to find the values of parameters (cost, capacity, travel time, etc.) which makes the given solution optimum and which are differ from the given parameters as little as possible.

Within the mathematical programming community, the interest in inverse optimization problems was generated by the paper due to Burton and Toint [14] in 1992, they consider a directed graph with a set of nonnegative costs on its arcs and modify these costs as little as possible to make the given path between the origin and destination, the shortest path. An important contribution on inverse optimization in the field of mathematical programming was Ahuja and Orlin [1]. They consider a general linear programming problem and prove that the inverse problem of linear program under l1 and l∞ norm is also linear program. Although Zhang and Liu [16, 5] has first been investigated the inverse linear programming problem and Huang and Liu [15] also obtained the same result, but the approach used by Ahuja and Orlin is more general and can be used to solve several inverse optimization problems.

**Inverse Optimization: Classification and Formulation**

Inverse optimization problem can be divided into two categories: inverse solution optimization problem and inverse objective value optimization problem.

1. **Inverse solution optimization problem**

In an inverse solution optimization problem, we have a desired solution and we wish to make it optimal by adjusting the parameters associated with the decision variables in the objective function or in the set of constraints, so that the adjusted values of parameters are differ from the given parameters as little as possible. These types of problems are known as inverse problems and most of the problems on inverse optimization, available in the literature are generally belongs to this group of problems. We have also considered the same type of problems in our work[10,13].

Let us consider the following optimization problem

min

s.t. (1)

where is the set of feasible solutions, a given cost vector. If is the given feasible solution, is the perturbed value of *c* and be some norm then the inverse problem to (1) is

min

s.t.

 (2)

1. **Inverse objective value optimization problem**

In an inverse objective value optimization problem, we do not have the desired solution in advance, but we do have the desired value of the objective function and we wish to adjust the parameter values which make the optimal objective value equal to its desired value and differ from the given values of parameters as little as possible. In order to exclude any ambiguity, we use the term “Reverse problem” for these problems and this type of problems have reported by Cai et al. [2] and Zhang et al. [8]. We have also considered the similar problem in our work.

If is the desired objective value of problem (1) then the inverse problem can be formulated as:

min

s.t.

 (3)

**1.12.1 Related problems**

There are several variants of inverse optimization problems, which have been reported by various researchers. We are briefly discussing some of them.

1. **Adjustment Problem**

This type of problem is reported by Libura [4]. It is basically a generalization of inverse problem, where a subset *F* of the set of feasible solution *S* is given and we wish to adjust the objective coefficients as little as possible so that the optimal solution of the adjusted problem belongs to *F*.

If we consider the optimization problem (1) as the forward problem then the adjustment problem can be formulated as:

min

s.t.

 , (4)

1. **Partial Inverse Problem**

In this type of problems, a partial solution is given and we wish to make a minimum adjustment in the cost vector so that the modified problem has a full solution which contains the partial solution and the full solution is also optimal for the problem. Yang [6], Yang and Zhang [7] have considered the partial inverse problems of assignment problem and minimum cut problem.

Let us consider a partial solution defined as, where , then the inverse problem to (1) is

min

s.t.

, (5)

1. **Reverse Problem with Prescribed Objective Function Range**

It is the generalization of inverse objective value optimization problem, in which, instead of a single objective value, a set of objective value or a range of objective value is given. Heuberger [3] have considered this type of problem in his survey.

Let be the prescribed range of the objective function, then the reverse problem to (1) is

min

s.t.

 (6)

1. **Reverse Problem with Budget constraints**

In this problem, we specify a solution, but instead of making it optimal, we wish to find the best improvement that does not exceed a certain budget *B*.

Let is the given solution, is the given budget, then the reverse problem to (1) with budget constraints is

min

s.t. (7)

**5**. **Improvement Problem with Budget constraints**

In this type of problem, we neither specify a solution that we want to improve nor a certain objective value that we want to reach, but a certain limitation on budget is given. If we consider the original problem (1) then the inverse problem with budget constraints is formulated as:

min

s.t.

 , (8)

This type of problem is reported in Heuberger [3].

1. **Inverse problem with given objective value**

In this type of problem, we have solution that we want to optimize and also have a certain objective value, we want to reach. If we consider the original problem (1) then the inverse problem can be formulated as

min

s.t.

 (9)

Where is the given solution and the desired objective value. Jain and Arya [10,13] proposed some inverse optimization models for this types of problem

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