**Innovative Solutions for Modeling and Analyzing Ebola Virus Dynamics**

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**ABSTRACT.** In the relentless pursuit of combating the Ebola virus, mathematical modeling stands as a powerful tool for understanding its dynamics and devising effective control strategies. This paper presents a novel approach to solve the Ebola virus model by integrating the Laplace transform method with a newly developed iterative technique. The proposed methodology offers a robust and efficient framework for analyzing the intricate dynamics of the virus spread and the efficacy of various intervention strategies.

**KEYWORDS:** Ebola virus model; Laplace Transform; Iterative technique; Epidemic dynamics.

1. **Introduction**

Fractional calculus [1], a branch of mathematics with roots tracing back centuries to luminaries like Leibniz and Euler, has become increasingly relevant in modern science and engineering. Initially conceived to extend traditional calculus to non-integer orders, fractional calculus has found applications in various fields, offering new insights and tools for understanding complex systems. Alongside this, advancements in computational techniques have transformed how we tackle mathematical problems. These numerical methods, powered by computers, provide efficient ways to approximate solutions to equations, making it easier to analyze dynamic systems and simulate real-world scenarios. The advent of computers and computational techniques has revolutionized the field of numerical methods, enabling researchers to tackle previously intractable problems in mathematics and science. Numerical methods play a pivotal role in approximating solutions to differential equations, offering efficient and accurate approaches for analyzing dynamical systems and simulating real-world phenomena. In recent years, there has been growing interest in leveraging numerical methods to solve fractional differential equations, capitalizing on their flexibility and computational power to address a wide range of applications, for details see [2–11].

The Ebola virus disease (EVD), characterized by its high mortality rate and potential for rapid spread, remains a significant global health concern. Since its discovery in 1976, numerous outbreaks have occurred, causing devastating impacts on affected communities and posing challenges for healthcare systems worldwide. Mathematical modeling has emerged as a valuable tool for understanding the dynamics of EVD transmission and assessing the effectiveness of intervention strategies aimed at controlling its spread [12–19].

In this paper, we present a novel approach to solving the Ebola virus model by integrating the Laplace transform method with a newly developed iterative technique. Our motivation stems from the need for more efficient and accurate methods to analyze the complex dynamics of EVD transmission and evaluate the impact of various control measures. The Laplace transform method offers an alternative approach by transforming the differential equations into algebraic equations,

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thereby simplifying the problem-solving process. By combining the strengths of the Laplace transform with innovative iterative techniques, we aim to address the challenges posed by nonlinearities and dynamic changes inherent in epidemic models. However, the application of the Laplace transform to epidemic models encounters challenges due to the presence of nonlinearities and non-constant coefficients [20–22]. To overcome these limitations, we introduce a new iterative technique tailored to address the specific characteristics of the Ebola virus model. This iterative approach iteratively refines the solution obtained through the Laplace transform method, enhancing accuracy and convergence properties.

This paper is organized in various sections as follows: In Section 1, we present the introduction. In the second Section, description of the model used in this study is given. important definitions that are used in this paper are given in section 3. In Section 4, description of the technique is given. In Section 5, application of the technique to the proposed model is given. In Section 6, numerical simulation and discussion is presented. Finally, Section 7 contains the conclusion.

1. **Description of the model**

The Ebola virus model represents a system of nonlinear differential equations that describe the interactions between susceptible, infected, and recovered individuals within a population. The Ebola virus model (EVD) is written as [12]:

with the initial conditions and is the order of the derivative.

Where N be the total population, S be the susceptible, I be the infected, R be the number of recovery and D be the number of disease induced deaths due to EVD. The description of the parameters is given in table 1.

1. **Mathematical preliminaries**

This section presents the basic definitions of Caputo fractional derivatives, Laplace transform and their properties that are used throughout the paper.

**Definition 2.1.** The Caputo fractional derivative ofis presented as [23]:

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where

**Definition 2.2.** The Laplace transform (LT) of a function is defined as [23]:

and the Inverse Laplace transform of is defined as:

**Definition 2.3.** The Laplace transform of the Caputo derivative is defined as [23]:

where

1. **Description of the technique**

In this section, we describe the proposed technique to solve the Ebola virus model. For this, we consider the fractional initial value problem with Caputo fractional derivative as:

 (1)

with the initial condition as:

where and represents the linear and non-linear operators respectively, and is any given function.

Now, apply Laplace transform on both sides, we get

Taking Inverse Laplace transform on both sides, we obtain

which is of the form of eq. (1).

Now, by solving the above equation using new iterative technique described in [24], we have the solution of eq. (1) in the form of infinite series as:

where

1. **Application**

In this section, we implemented the proposed technique on fractional Ebola virus model given by:

with the initial conditions and is the order of the derivative.

After applying the technique described in section 3, we get the solution of the above equation in the form of series as:

where , , , ,

and so on. So, we get the approximate solution of EVM with three iterations as:

1. **Simulation and discussion**

Numerical simulations are obtained for and at distinct values of order of derivative using Matlab software and values of parameter are given in table 1. Fig. 1a represents the behavior of susceptible with time for different values of order of derivative. It shows that the number of susceptible decreases as the order of derivative decreases. Fig. 1b represents the behavior of infected with time for different values of order of derivative which shows that the number of infected increases with decrease in order of derivative. Similar behavior is seen from fig. 1c and 1d, which represents the behavior of recovered and deaths respectively, with time for different values of order of derivative. Both recovered and number of deaths are increasing with decreasing of order of derivative.

**Table 1. Description and values of parameters** [25]**.**

|  |  |  |
| --- | --- | --- |
| Parameters | Description | Values |
|  | Rate of Infection | 0.01 |
|  | Rate of Susceptible | 0.02 |
|  | Death rate due to disease | 0.6 |
|  | Death rate due to other reason | 0.01 |
|  | Rate of Recovery | 0.4 |
|  | Total population | 1000 |
|  | Number of susceptible at | 900 |
|  | Number of infected at | 10 |
|  | Number of recovered at | 0 |
|  | Number of deaths at | 0 |

**Table 1.** Description of the parameters.

 (a) (b)

(c) (d)

**Figure 1.** Behavior of **(a)** Susceptibles, **(b)** Infected, **(c)** Recovered and **(d)** Deaths with time at distinct values of .

1. **Conclusion**

In this article, the application of a hybrid technique on the Ebola virus model is given. the combination of fractional calculus, numerical methods, and mathematical modeling offers a potent approach for understanding and controlling the transmission dynamics of infectious diseases like Ebola virus disease. Through innovative techniques such as the integration of the Laplace transform and iterative methods, we can improve the accuracy and efficiency of modeling efforts, ultimately leading to more effective strategies for disease prevention and control. Continued research and collaboration in this interdisciplinary field hold promise for mitigating the impact of epidemics on public health and society.

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