Generalized Ulam-Hyers Stability Of System Of Quadratic Functional Equations Originating From Geometrical Equations

E. Sathya

Department of Mathematics

Kalaignar Karunanidhi Government Arts College, Tiruvanamalai – 606 603, Tamil Nadu, India.

e-mail: sathy24mathematics@gmail.com

T. Namachivayam

Department of Mathematics

Kalaignar Karunanidhi Government Arts College, Tiruvanamalai – 606 603, Tamil Nadu, India.

e-mail: namachi.siva1968@gmail.com

R. Suganya

Department of Physics

Kalaignar Karunanidhi Government Arts College, Tiruvanamalai – 606 603, Tamil Nadu, India.

e-mail: suganya.tvm@gmail.com

ABSTRACT

In this paper, the authors established the generalized Ulam-Hyers stability of system of quadratic functional equations which originating from Leibniz formula in Euclidean Geometry and median of a triangle in intuitionistic fuzzy Banach space using Hyers direct method.

Keywords—Quadratic functional equations, generalized Ulam – Hyers stability, intuitionistic fuzzy Banach space, Hyers method, Geometry, Leibniz formula in Euclidean Geometry, median of a triangle.

# INTRODUCTION

The revision of stability problems for functional equations is coupled to a query of Ulam [36,37] concerning the stability of group homomorphisms and confidently responded for Banach spaces by Hyers [20]. It was supplementary generalized and outstanding results was attained by number of authors see ([3,18,26,29,32]). The general solution and generalized Ulam-Hyers-Rassias stability of quadratic functional equation was investigated by Cholewa [15], S. Czerwik [16], Jung [23]. Ravi [30,31].

During the last eight decades, the overhead problems was attempted by numerous authors and its solutions via various forms of functional equations were discussed one can refer [1-2,4-14,17,21-22,24-25,27-28] and references cited there in.

Geometry is a branch of [mathematics](https://en.wikipedia.org/wiki/Mathematics) concerned with properties of space such as the distance, shape, size, and relative position of figures. Geometry is, along with [arithmetic](https://en.wikipedia.org/wiki/Arithmetic), one of the oldest branches of mathematics. A mathematician who works in the field of geometry is called a [geometer](https://en.wikipedia.org/wiki/List_of_geometers). Until the 19th century, geometry was almost exclusively devoted to [Euclidean geometry](https://en.wikipedia.org/wiki/Euclidean_geometry), which includes the notions of [point](https://en.wikipedia.org/wiki/Point_(geometry)), [line](https://en.wikipedia.org/wiki/Line_(geometry)), [plane](https://en.wikipedia.org/wiki/Plane_(geometry)), [distance](https://en.wikipedia.org/wiki/Distance), [angle](https://en.wikipedia.org/wiki/Angle), [surface](https://en.wikipedia.org/wiki/Surface_(mathematics)), and [curve](https://en.wikipedia.org/wiki/Curve), as fundamental concepts.

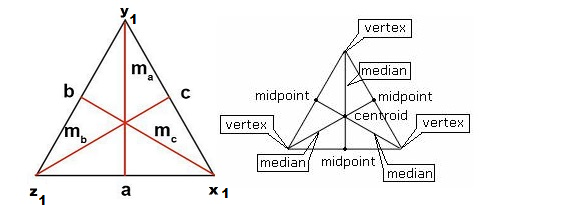
Originally developed to model the physical world, geometry has applications in almost all [sciences](https://en.wikipedia.org/wiki/Science), and also in [art](https://en.wikipedia.org/wiki/Art), [architecture](https://en.wikipedia.org/wiki/Architecture), and other activities that are related to [graphics](https://en.wikipedia.org/wiki/Graphics). Geometry also has applications in areas of mathematics that are apparently unrelated. The geometry that underlies [general relativity](https://en.wikipedia.org/wiki/General_relativity) is a famous application of non-Euclidean geometry.

During and late the 19th century several discoveries enlarged dramatically the scope of geometry has been greatly expanded, and the field has been split in many subfields that depend on the underlying methods [differential geometry](https://en.wikipedia.org/wiki/Differential_geometry), [algebraic geometry](https://en.wikipedia.org/wiki/Algebraic_geometry), [computational geometry](https://en.wikipedia.org/wiki/Computational_geometry), [algebraic topology](https://en.wikipedia.org/wiki/Algebraic_topology), [discrete geometry](https://en.wikipedia.org/wiki/Discrete_geometry). Also, the properties of Euclidean spaces that are disregarded [projective geometry](https://en.wikipedia.org/wiki/Projective_geometry) that consider only alignment of points but not distance and parallelism, [affine geometry](https://en.wikipedia.org/wiki/Affine_geometry) that omits the concept of angle and distance, [finite geometry](https://en.wikipedia.org/wiki/Finite_geometry) that omits [continuity](https://en.wikipedia.org/wiki/Continuity_(mathematics)), and others. This enlargement of the scope of geometry led to a change of meaning of the word space, which originally referred to the three-dimensional [space](https://en.wikipedia.org/wiki/Space) of the physical world and its [model](https://en.wikipedia.org/wiki/Model) provided by Euclidean geometry; presently a geometric space, or simply a space is a [mathematical structure](https://en.wikipedia.org/wiki/Mathematical_structure) on which some geometry is defined. For more detailed see [38].

The main aim of this paper is to convert the geometrical properties into functional equations which satisfies the properties via its solutions.

**A MEDIAN OF A TRIANGLE**

In geometry, a median of a triangle is a line segment joining a vertex to the midpoint of the opposing side. Every triangle has exactly three medians: one running from each vertex to the opposite side. In the case of isosceles and equilateral triangles, a median bisects any angle at a vertex whose two adjacent sides are equal in length.



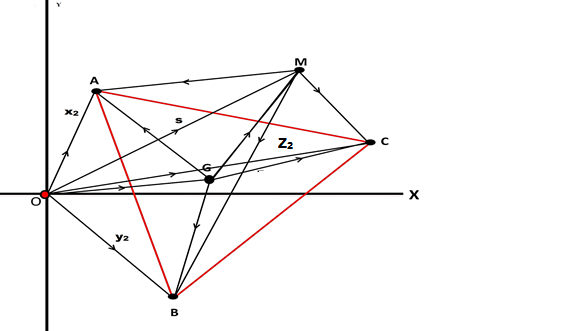
In a above triangle with the sides  and  the median drawn to the side  has the length of

. (1.1)

**B. LEIBNIZ QUADRATIC FORMULA IN EUCLIDEAN GEOMETRY**

Let be an arbitrary point lying on the plane of the triangle  and  is the centroid (= Gravity center) of, then

. (1.2)



The above equation (1.2), can be transformed into quadratic functional equation of the median from  is given by

 (1.3)

Also, equation (1.4), can be transformed into quadratic functional equation of the centroid  is set by

 (1.4)

In this paper, the authors established the generalized Ulam-Hyers stability of system of quadratic functional equations (1.3) and (1.4) which originating from Leibniz formula in Euclidean Geometry and median of a triangle in intuitionistic fuzzy Banach space using Hyers direct method.

**II BASIC DEFINITIONS RELATED TO INTUITIONISTIC FUZZY NORMED SPACES**

In this section, we provide some basic definitions and notations related to intuitionistic fuzzy normed spaces as in [34,35,36].

**Definition 2.1.** A binary operation \* :  is said to be a continuous t-norm if it satisfies the following conditions :

 \* is associative and commutative,

 \* is continuous,

  for all ,

  whenever and for each 

**Definition 2.2.** A binary operation  :  is said to be a continuous t-conorm if it satisfies the following conditions :

 is associative and commutative,

 is continuous,

  for all ,

  whenever and for each 

**Definition 2.3.** The five-tuple is said to be an intuitionistic fuzzy normed space (for short, IFNS) if is a vector space, \* is a continuous t-norm, is a continuous t-conorm,  are fuzzy sets on  such that for all and satisfying the following conditions :

[IFNS1] 

[IFNS2] 

[IFNS3]  if and only if 

[IFNS4]  for each 

[IFNS5] 

[IFNS6] is continuous,

[IFNS7] and

[IFNS8] 

[IFNS9]  if and only if 

[IFNS10]  for each 

[IFNS11] 

[IFNS12] is continuous,

[IFNS13] and.

In this case  is called an intuitionistic fuzzy norm

**Example 2.3.** Let  be a normed space,  and **** for all  For all , , , consider  and  then is an IFNS.

**Definition 2.5.** Let be an IFNS. Then, a sequence is said to be intuitionistic fuzzy convergent to if and  for all .

**Definition 2.6.** Let be an IFNS. Then, a sequence is said to be intuitionistic fuzzy Cauchy sequence if and for all and 

**Definition 2.7.** Let be an IFNS. Then,  is said to be Banach space if every intuitionistic fuzzy convergent in .

**III INTUITIONISTIC FUZZY STABILITY**

From now on  be an Intuitionistic Fuzzy normed space  be an Intuitionistic Fuzzy Banach space respectively. In this section, using an idea of Gavruta. We prove the stability of in the spirit of Hyers, Ulam and Rassias. For convenience we use the following abbreviation for a given mapping and  by



and

for all .

**Theorem 3.1 :** Let  be fixed and let  are mappings for some  with  satisfying the conditions

 (3.1)

  (3.2)

and  are functions satisfying the inequalities

 (3.3)

for all  and all  Then there exists a unique quadratic mappings  satisfying (1.3) and (1.4) such that

 (3.4)

for all and all  The mappings  are obtained by

 (3.5)

for all .

**Proof :** Assume . Replacing  by  in (3.3), using evenness of  and [IFNS4], one can get

 (3.6)

for all  and all . Replacing  by  in (3.6), one can obtain

 (3.7)

for all  and all . Using [IFNS4], (3.1) in (3.7), one can arrive

 (3.8)

for all  and all . Replacing  by  in (3.8), one can have

 (3.9)

for all  and all . One can easy to see verify that

 (3.10)

for all . From equations (3.9) and (3.10), we have.

 (3.11)

for all and  where 

for all and all . Replacing  by  in (3.11) and using (3.1), [IFNS4], one can obtain

 (3.12)

for all and all  and all . Replacing  by  in (3.12), one can get

 (3.13)

for all and all  and all  . Using [IFNS4] in (3.13), one can obtain

 (3.14)

for all and all  and all .

Since  and  the Cauchy Criterion for convergence in IFNS, it shows that



are Cauchy sequences in  and it is complete, this sequences converges to some point . So, one can we define the mapping  by



for all . Letting  in (3.14), one can get

 (3.15)

for all and all  Letting  in (3.15) and using [IFNS6], one can arrive.

 (3.16)

for all and .

Now, we need to prove  satisfies (1.3) and  satisfies (1.4), replacing  by in (3.3) respectively, one can obtain

 (3.18)

(3.19)

for all and all  Now





Letting  in (3.18) and using (IFN7), (IFN13), (IFN3), (IFN9), we can see that  satisfies (1.3).

Similarly, we can prove the another result.

To prove the uniqueness of , assume there exists a mapping  satisfies (1.3), satisfies (1.4) and (3.4). Hence,

 (3.20)

for all  and all . Since,



for all  and by [IFNS5] and [IFNS13], one can arrive



for alland all . Therefore



for all and all .Hence,



Therefore  are unique. So, the proof holds for .

Now,  by  in (3.6) and using [IFNS4], one can get

 (3.21)

for alland all . The rest of the proof is analogous to that of prior case. So, the proof holds for. This concludes the proof of the theorem.

**Corollary 3.2.** Assume  are functions satisfying the inequalities

 (3.22)

for all  and all  Then there exists a unique quadratic mappings  satisfying (1.7) such that



 (3.23)

for all and all 

**VI CONCLUSIONS**

In this paper, we establish the geometrical mathematical properties convert into quadratic functional equations and analyzed the generalized Ulam-Hyers stability of the functional equations in intuitionistic fuzzy Banach space using Hyers direct method.

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